Volume 65, Issue 9, 2021
Journal of Scientific Research
of
The Banaras Hindu University

An Evolutionary Based Optimization for Price Varying Demand Inventory Model with Linear Holding Cost

Sandesh Kurade

Department of Statistics, MES Abasaheb Garware College, Pune 411004, India
sandeshkurade@gmail.com

Abstract: **Supply chain is a set of interlinked facility such as: supplier, producer, manufacturer, retailer, customer etc. Each of these entities in a supply chain are interconnected and performed independently in such a way that, profit of the chain is high or low cost. Optimization of transportation and inventory or both simultaneously in a supply chain are the two mostly crucial problems focused by authors in the literature. Due to the importance of inventory optimization in a supply chain, here a stochastic inventory policy for perishable items is developed. Model is developed by assuming cost of sale sensitive market demand, continuous time varying holding cost, and exponential distributed deterioration under partial shortages with time dependent backlog rate. The formulated model will be useful for single as well as multi tier food processing supply chains. Profit function of the model is formed by considering various cost components of the model. The developed model is continuous, twice differentiable but highly non-linear hence, the numerical illustrations are solved using proposed genetic algorithm. The over and under effect of various parameters on the developed model is checked using sensitivity analysis.**

Index Terms: **Exponential deterioration, genetic algorithm, linear holding cost, perishable, random demand, supply chain.**

I. Introduction

Inventories are the physical stock of deteriorated or non-deteriorated goods or items. In an industry the inventory needs to be maintained well such that, the profit of the industry is maximum or loss is minimum. Economic order quantity (EOQ) inventory models with constant demand for a perishable item have been studied by several authors in literature. But in reality, demand of such items is not constant. It is affected by several external factors. In business, pricing is the most significant factor to attract pool of customers. In inventory optimization, it is the most important factor to success in business (Maihami and Kamalabadi (2012)). It is general tendency that lower the price of a perishable item, higher the demand pattern (Wu et al. (2017)). It is an important factor which significantly affects demand rate of the deteriorated item. Hence, market demand is critically unfair by cost of sale of the perishable unit. Such type of demand pattern was generally seen in food, garment, pharmaceutical, and cosmetic industries. Deterioration means item is not up to the mark of quality. Spoilage, expired, damaged etc. are some of the cases of deterioration. Also, when new technology or items are available then sell of old items decreases over time. This is also considered as deterioration. During normal storage period, several items such as pharmaceutical items, food items, perfumes, blood etc. decrease their quality under deterioration and their life is fixed. Hence, while deciding the optimum level of inventory for a single tier supply chain, the marginal loss due to decline in their quality of use cannot be unnoticed. Medical items, bakery items, meat, seafood, vegetables, dairy products, cement, etc. are some of the examples of perishable items. The various continuous probability distributions with known parameter values for instance, exponential, two-parameter Weibull, three-parameter Weibull, and Gamma were considered to model the deterioration of the perishable item.

The foremost goal of the paper is the formulation of an economic order quantity (EOQ) inventory model with cost of sale responsive market demand of the deteriorated item, variable deterioration, holding cost as a continuous and increasing function of time under partial backlogging. Also under these assumptions, the most suitable and economical backlog rate for the inventory system in a single tier supply chain is obtained. Due to these realistic assumptions the formulated model has several applications. Organization of the article as: The detailed review of literature will be explained in Section II. Mathematical model will be explained in Section III. Also, assumptions with notations considered for the formulation of mathematical model will be explained in the same Section. The binary coded genetic algorithm for solving purpose presented in Section IV. Results and discussions of the numerical examples presented in Section V. The effect of various parameters of the model on the profit expression using sensitivity analysis is discussed in Section VI. Finally, concluding remarks of the article are presented.

II. REVIEW OF LITERATURE

The optimal pricing inventory models were developed by Dye et al. (2007), Abad (1996, 2001), and Chang et al. (2006). Alfares (2007) formulated an inventory model under variable holding cost with stock responsive market demand. Cost of sale sensitize demand model was proposed by Das et al. (2020) under partial backlogging and it was solved by using three different Particle Swarm Optimizations (PSOs). For reducing the deterioration effect, preservation of items technique was used to develop the model. Maihami & Kamalabadi (2012) formulated a replenishment policy for an inventory system with price and time dependent demand of decay items. Shortages were partially backlogged of the model. Panda et al. (2019) proposed an inventory model under partial backlogging with stock, price and frequency of advertisement dependent demand rate with constant deterioration. Sensitivity analysis was performed to check the impact of various parameters on the developed model. Each parameter was changed from -20% and +20% by fixing other parameters. Kurade & Latpate (2021) developed new genetic algorithms for solving time dependent demand and variable deterioration rate inventory models. No, complete and partial backlog situations were considered in the model.

An EOQ inventory model was formed by Sana (2010) and it was enriched with cost of sale sensitive demand and time varying deterioration by allowing partially backlogged shortages under two different time-dependent backlog rates. Dye and Hsieh et al. (2007) formed an EOQ model under price and stock dependent demand with varying rate of deterioration. In the proposed replenishment policy, partially backlogged shortages were considered. Wee (1997) proposed an inventory replenishment policy with price dependent demand and varying deterioration rate. Dye (2012) formulated a deterministic EOQ inventory model under finite horizon with price and time varying demand rate. The profit function dependent on number of replenishments stock, price, and time were developed and solved by using PSO. Wu et al. (2017) formulated an inventory model with price sensitive demand. In which deterioration rate was dependent on the expiration rate of the perishable item. Li & Teng (2018) developed an inventory model for a perishable item with demand depending on selling cost, reference price, freshness condition, and stock. Ruidas et al. (2020) formulated a deterministic economic product quantity (EPQ) inventory model. Level of inventory and selling cost sensitive demand, probabilistic rate of production were assumed in the developed model. Complete backlogged situation were considered while solving the numerical examples. Sharma et al. (2015) developed an EPQ inventory model with price dependent demand under partially backlogged shortages. Time dependent deterioration and demand rate dependent production rate were also assumed for the development of model. Sridevi et al. (2010) proposed a deterministic EOQ inventory model under Weibull distributed production rate and price of a perishable item sensitive market demand. Sensitivity analysis of various parameters of the profit expression was performed. The model is applicable in food, cereals, edible oil and petrochemical industries. Valliathal & Uthayakumar (2011) developed a deterministic EOQ inventory model by assuming selling price and rebate value dependent demand rate. Shortages were allowed and the unsatisfied market demand was partially backlogged.

An inventory model useful for multi tier supply chain involving supplier, retailer and customer is developed by Li et al. (2019). Selling price, expiration date and credit period varying demand were considered in the developed model. Dey et al. (2019) formulated an integrated inventory model for single tier supply chain with selling price dependent demand rate. Demand during lead time follows a Poisson distribution by considering environmental impacts was assumed in the model. Shortages were permitted in the developed model and they were completely backlogged. Buyer and vendor profit was calculated independently, to maximize the total profit of single tier supply chain. Jaggi et al. (2014) formulated an inventory model in two warehouse scenario. The model was developed by assuming selling price dependent demand rate under complete backlogging. Geetha & Udayakumar (2016) formulated an EOQ optimal inventory policy for perishable items. Selling price and advertisement dependent demand with salvage value of perishable items under partial backlogged situation were assumed in the model. To know behavior of the model, effect of various key parameters of the model were studied through sensitivity analysis. Latpate & Kurade (2017) formulated a new evolutionary algorithm using fuzzy set theory and genetic algorithm useful for solving multi-objective optimization problem occurs in multi tier supply chain. For optimizing crude oil supply chain of India, Latpate & Kurade (2020) proposed a new hybrid optimization algorithm using non-dominated sorting approach. The superiority of the algorithm was checked for the real world data.

Pal et al. (2006) developed an inventory model by considering selling cost, time and frequency of advertisement dependent market demand with constant lead time under partial backlogging situation. Bhunia et al. (2015) formulated an inventory model for deteriorating items with two independent storage facilities of a firm and constant lead time. Selling price, time and frequency of advertisement dependent market demand rate were assumed in the model with partial shortages. Fuzzy inventory model by taking into consideration the shortage follows inventory policy proposed by Shaikh et al. (2018). Shortages were partially backlogged in the formulated model.

Selling price and frequency of advertisement dependent demand rate with permissible delay in payments were assumed in the model. Ghoreishi et al. (2015) developed a deterministic inventory model for non-instantaneous perishable items. Price and inflation dependent demand rate were assumed in the model. For settling the financial account, delay in payments with partial backlogging situation was assumed. Due to competitive nature of the market, the return policy and time value of money were also considered while development of the inventory model. Sahoo et al. (2019) formulated an inventory model under linear time dependent deterioration rate under partially backlogged situation. De & Sana (2015) developed a fuzzy EOQ model. Triangular membership function was used to represent the uncertainty among the parameters of the model. Selling cost with effort of promotion dependent market demand was assumed in the formulated model. From the above extensive survey, we get clear idea that price dependent demand with holding cost as a continuous increasing function of time is not considered by the authors while developing the inventory models.

III. MATHEMATICAL MODEL

In this study, an EOQ inventory model by considering price varying demand, exponential deterioration and time dependent linear holding cost is developed. The variation in inventory during inventory cycle under partial backlogging is shown in Figure 1. Inventory is decreases from level S due to satisfying market demand and deterioration in $[0, t_1]$. In $[t_1, T]$, market demand is partially backlogged with two different backlog rates as, i) $B(T-t) = 1/(1+\delta(T-t))$ and
ii) $B(T-t) = exp[-\delta(T-t)]$.

A. Assumptions

1. Model is developed for single perishable item.
2. The rate of deterioration is constant i.e., exponential deterioration with rate θ.
3. Selling price dependent demand i.e., $D(P) = (a - b P)$, $a > 0, b > 0$.
4. Negligible lead time and infinite rate of replenishment.
5. Partially backlogged shortages.
6. Time dependent linear holding cost i.e., $H(t) = \alpha + \beta t;\ \alpha, \beta > 0$.

B. Notations

- Q, economic order quantity (EOQ).
- S, maximum inventory level.
- $I(t)$, level of inventory at time t, $0 < t < T$.
- $g(t)$, deterioration rate of the perishable item follows an exponential distribution i.e., $g(t) = \theta\ (> 0)$
- $D(P)$, demand rate.
- a, b, demand rate parameters ($a > 0, b > 0$).
- $H(t)$, holding cost.
- $\alpha, \beta > 0$, holding cost parameters.

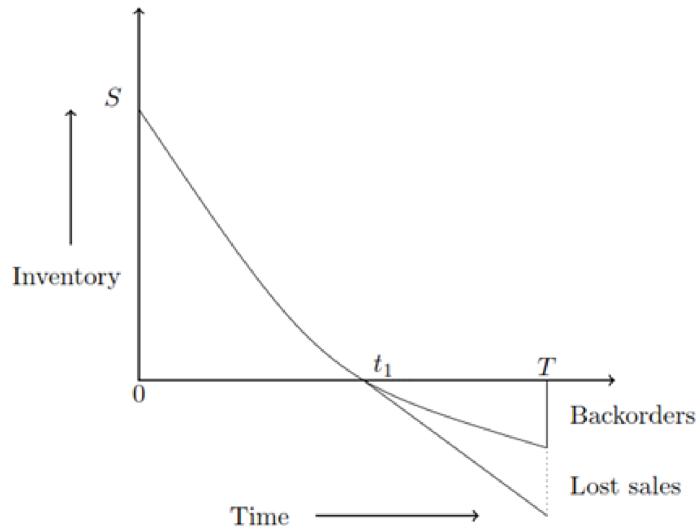

Fig. 1. Behavior of inventory in partial backlogging

- C_{Ord}, order cost.
- P, selling cost.
- C, purchase cost.
- C_1, deterioration cost.
- C_2, shortage cost.
- C_3, opportunity cost.
- $\pi(t_1, T)$, profit expression.
- $\delta(> 0)$- backlogging parameter.
(All costs are per unit in $).

Decision variable:

- T, the length of an inventory cycle.
- t_1, the length of time after that shortage will start.

Let us consider, case i) $\mathbf{B(T-t) = 1/(1+\delta(T-t))}$

Then, the behavior of inventory is represented by equations (1) and (2) as,

$$\frac{d\,I(t)}{dt} + g(t)\,I(t) = -D(P);\ 0 \le t \le t_1 \quad (1)$$

$$\frac{d\,I(t)}{dt} = -\frac{D(P)}{1+\delta(T-t)};\ t_1 \le t \le T \quad (2)$$

Boundary conditions, $I(0) = S$; and $I(t_1) = 0$.

Thus, at the above boundary conditions solutions of equations (1) and (2) are,

$$I(t) = Se^{-\theta t} - D(P)e^{-\theta t}\int_0^t e^{\theta x}dx \ ;\ 0 \le t \le t_1 \quad (3)$$

$$I(t) = -D(P)\int_{t_1}^t B(T-x)\ dx;\ \ t_1 \le t \le T \quad (4)$$

In equation (3) put $t = t_1$ and using boundary condition, we get

$$I(t_1) = Se^{-\theta t_1} - D(P)e^{-\theta t_1}\int_0^{t_1} e^{\theta x}dx$$

After simplifying, we get

$$S = \left(\frac{a-bP}{\theta}\right)\left(e^{\theta t_1} - 1\right) \quad (5)$$

Thus, the economic order quantity Q becomes,

$$Q = S + \int_{t_1}^{T} \frac{D(P)}{1+\delta(T-x)}\, dx$$

$$= (a - bP)\left[\frac{(e^{\theta t_1}-1)}{\theta} + \int_{t_1}^{T} \frac{D(P)}{1+\delta(T-x)}\, dx\right] \quad (6)$$

$$\text{sales revenue} = \int_0^{t_1} D(P)\, dx + \int_{t_1}^{T} \frac{D(P)}{1+\delta(T-x)}\, dx$$

$$= (a - Pb)P\left[t_1 + \int_{t_1}^{T} \frac{1}{1+\delta(T-x)}\, dx\right] \quad (7)$$

Then,

$$\text{total amount of deteriorated items} = S - \int_0^{t_1} D(P)\, dx$$

$$= \frac{(a-Pb)}{\theta}\left[e^{\theta t_1} - 1 - \theta t_1\right]$$

Therefore,

$$\text{deterioration cost} = C_1 * \text{amount of deteriorated items}$$

$$= C_1 \frac{(a-Pb)}{\theta}\left[e^{\theta t_1} - 1 - \theta t_1\right] \quad (8)$$

The amount of shortage during $[t_1, T]$ is,

$$\text{amount of shortage} = -\int_{t_1}^{T}\left[\int_{t_1}^{t} \frac{-D(P)}{1+\delta(T-x)}\, dx\right] dt$$

$$= D(P)\int_{t_1}^{T} \frac{T-t}{1+\delta(T-t)}\, dt$$

$$\text{Shortage cost} = C_2 * \text{amount of shortage}$$

$$= C_2(a - Pb)\int_{t_1}^{T} \frac{T-t}{1+\delta(T-t)}\, dt \quad (9)$$

The amount of lost sales during $[t_1, T]$ is,

$$\text{amount of lost sales} = D(P)\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right] dt$$

$$= (a - Pb)\delta\int_{t_1}^{T} \frac{T-t}{1+\delta(T-t)}\, dt$$

The opportunity cost during $[t_1, T]$ is,

$$\text{opportunity cost} = C_3 * \text{amount of lost sales}$$

$$= C_3(a - Pb)\delta\int_{t_1}^{T} \frac{T-t}{1+\delta(T-t)}\, dt \quad (10)$$

$$\text{Material cost} = C\left[\int_0^{t_1} I(t)\, dt - \int_{t_1}^{T} I(t)\, dt\right]$$

$$= C\left[\int_0^{t_1}\left[Se^{-\theta t} - D(P)e^{-\theta t}\int_0^{t} e^{\theta x}dx\right] dt - \int_{t_1}^{T}\left[-D(P)\int_{t_1}^{t} \frac{1}{1+\delta(T-x)}\, dx\right] dt\right]$$

$$= (a - Pb)C\left[\begin{array}{c}\frac{(e^{-\theta t_1}-1)(1-e^{\theta t_1})}{\theta} \\ -\frac{\theta t_1 + (e^{-\theta t_1}-1)}{\theta^2} + \int_{t_1}^{T} \frac{T-t}{1+\delta(T-t)}\, dt\end{array}\right] \quad (11)$$

$$\text{Holding cost} = C\left[\int_0^{t_1}(\alpha + \beta t)I(t)\, dt\right]$$

$$= C\left[\alpha\int_0^{t_1} I(t)\, dt + \beta\int_0^{t_1} t\, I(t)\, dt\right]$$

$$= C\left[\begin{array}{c}\alpha\int_0^{t_1}\left[Se^{-\theta t} - D(P)e^{-\theta t}\int_0^{t} e^{\theta x}dx\right] dt \\ +\beta\int_0^{t_1} t\left[Se^{-\theta t} - D(P)e^{-\theta t}\int_0^{t} e^{\theta x}dx\right] dt\end{array}\right]$$

$$= \frac{(a-Pb)C}{\theta^2}\left[\begin{array}{c}\alpha(1-e^{\theta t_1})(e^{-\theta t_1}-1) \\ -\alpha\left[(e^{-\theta t_1}-1) + t_1\theta\right] \\ -\beta(1-e^{\theta t_1})(e^{-\theta t_1}-1)\left[t_1 + \frac{1}{\theta}\right] \\ -\beta\left[\begin{array}{c}\theta\frac{t_1^2}{2} + t_1(e^{-\theta t_1}-1) \\ -\frac{1}{\theta}(e^{-\theta t_1}-1)\end{array}\right]\end{array}\right] \quad (12)$$

Therefore profit per unit time is,

$$\pi(t_1, T) = \frac{1}{T}\left(\begin{array}{c}\text{Sales revenue} - \text{Deterioration cost} \\ -\text{Shortage cost} - \text{Opportunity cost} \\ -\text{Material cost} - \text{Holding cost} \\ -\text{Order cost}\end{array}\right)$$

By substituting equations (7) – (12) we get,

$$\pi(t_1, T) = \frac{1}{T}\left(\begin{array}{c}(a - Pb)P\left[t_1 + \int_{t_1}^{T}\frac{1}{1+\delta(T-x)}\, dx\right] \\ -C_1\frac{(a-Pb)}{\theta}\left[e^{\theta t_1} - 1 - \theta t_1\right] \\ -C_2(a - Pb)\int_{t_1}^{T}\frac{T-t}{1+\delta(T-t)}\, dt \\ -C_3(a - Pb)\delta\int_{t_1}^{T}\frac{T-t}{1+\delta(T-t)}\, dt - \\ (a - Pb)C\left[\begin{array}{c}\frac{(e^{-\theta t_1}-1)(1-e^{\theta t_1})}{\theta} \\ -\frac{\theta t_1 + (e^{-\theta t_1}-1)}{\theta^2} \\ +\int_{t_1}^{T}\frac{T-t}{1+\delta(T-t)}\, dt\end{array}\right] - \\ \frac{(a-Pb)C}{\theta^2}\left[\begin{array}{c}\alpha(1-e^{\theta t_1})(e^{-\theta t_1}-1) \\ -\alpha\left[(e^{-\theta t_1}-1) + t_1\theta\right] \\ -\beta(1-e^{\theta t_1})(e^{-\theta t_1}-1)\left[t_1 + \frac{1}{\theta}\right] \\ -\beta\left[\begin{array}{c}\theta\frac{t_1^2}{2} + t_1(e^{-\theta t_1}-1) \\ -\frac{1}{\theta}(e^{-\theta t_1}-1)\end{array}\right]\end{array}\right] \\ -C_{Ord}\end{array}\right) \quad (13)$$

Case ii) $B(T-t) = \exp[-\delta(T-t)]$

By substituting equations (A.1) – (A.7) explained in Appendix, we get the following profit expression.

$$\pi(t_1,T) = \frac{1}{T}\left(\begin{array}{c} (a-Pb)P\left[t_1+\int_{t_1}^{T} exp[-\delta(T-\mathrm{x})]\,dx\right] \\ -C_1\frac{(a-Pb)}{\theta}[e^{\theta t_1}-1-\theta t_1] \\ -C_2(a-Pb)\int_{t_1}^{T}\left[\int_{t_1}^{t} exp[-\delta(T-x)]\,\,dx\right]dt - \\ -C_3(a-Pb)\int_{t_1}^{T}[1-exp[-\delta(T-\mathrm{t})]]\,dt \\ -(a-Pb)C\left[\begin{array}{c}\frac{(e^{-\theta t_1}-1)(1-e^{\theta t_1})}{\theta} \\ -\frac{\theta t_1+(e^{-\theta t_1}-1)}{\theta^2} \\ +\int_{t_1}^{T}\left[\int_{t_1}^{t} exp[-\delta(T-x)]\,\,dx\right]dt\end{array}\right] \\ -\frac{(a-Pb)C}{\theta^2}\left[\begin{array}{c} \alpha(1-e^{\theta t_1})(e^{-\theta t_1}-1) \\ -\alpha[(e^{-\theta t_1}-1)+t_1\theta] \\ -\beta(1-e^{\theta t_1})(e^{-\theta t_1}-1)\left[t_1+\frac{1}{\theta}\right] \\ -\beta\left[\begin{array}{c}\theta\frac{t_1^2}{2}+t_1(e^{-\theta t_1}-1) \\ -\frac{1}{\theta}(e^{-\theta t_1}-1)\end{array}\right]\end{array}\right] \\ -C_{Ord}\end{array}\right) \tag{14}$$

Now, the optimization problem is to,

$$\underset{t_1,\,T}{\text{maximize}}\ \pi(t_1,T) \tag{15}$$

Subject to; $0 \le t_1 \le T$

IV. GENETIC ALGORITHM

It is an optimization algorithm inspired by evolution and it was invented by John Holland in 1975. The first step in the implementation of GA is to generate population of chromosomes. After initializing the population, each chromosome is then assessed and assigned a fitness value. It has two types viz., real coded genetic algorithm (RCGA) and binary coded genetic algorithm (BCGA). The RCGA works more efficiently with real numbers and when the dimension of the problem is sufficiently large. But when the dimension is too small then RCGA is not efficient. For smaller set of decision variables, BCGA works more efficiently than RCGA. Hence, in the present study optimal solution of the formulated optimization problem in (15) is obtained using BCGA, since the formulated EOQ inventory model has only two decision variables. The step by step procedure of BCGA is explained in Algorithm 1. For BCGA, roulette wheel selection methods works well than other selection methods available in the literature (Kurade and Latpate (2021)).

Algorithm 1

1. Initialize parameters.
2. Initialize population of binary strings.
3. Compute T_j and t_j for jth string as,
$$T_j = T_{j,min} + \frac{T_{j,max}-T_{j,min}}{2^{l_i}-1} * DCV(S_j)$$
, where $T_{j,max}$ and $T_{j,min}$ possible maximum and minimum value of time
$$t_j = t_{j,min} + \frac{t_{j,max}-t_{j,min}}{2^{l_i}-1} * DCV(S_j)$$
, where $t_{j,min}$ be the possible minimum value of time $t_{j,max} = T_j$, l_i, be the string length, and $DCV(S_j)$ be the decoded value of j[th] string.
4. Compute fitness function of each string.
5. Repeat the following steps:
    - Roulette Wheel Selection
    - One point Crossover
    - Bit wise Mutation
6. End

V. NUMERICAL EXAMPLE

Example 1-For case i) $\mathbf{B(T-t) = 1/(1+\delta(T-t))}$

For illustration purpose, following parameters are considered: C_{Ord}=\$250, C=\$10, C_1=\$9, C_2=\$9, C_3=\$5, P=\$25, $\alpha = 10$, $\beta = 2$, $\theta = 0.8$, $D(P) = 25 - 0.1\,P$, $\delta = 2$.

BCGA parameters are: P_{Cross} = 0.8, P_{Mut} = 0.2, string length = 100, population size = 100, number of iterations = 100.

Thus the formulated model gives the optimal result as,

Profit = \$ 271.97, EOQ (Q) = 17.34, S = 13.02, T = 0.7096 year, t_1= 0.4755 year, Deterioration cost = \$20.88, Shortage cost = \$7.10, Opportunity cost = \$4.73, Material cost = \$23.04 , Holding cost = \$-123.64

Example 2- For case ii) $\mathbf{B(T-t) = exp[-\delta(T-t)]}$

For illustration purpose, following parameters are considered: C_{Ord}=\$250, C=\$10, C_1=\$9, C_2=\$9, C_3=\$5, P=\$25, $\alpha = 10$, $\beta = 2$, $\theta = 0.8$, $D(P) = 25 - 0.1\,P$, $\delta = 2$.

BCGA parameters are: PCross = 0.8, PMut = 0.2, string length = 100, population size = 100, number of iterations = 100.

Thus the formulated model gives the optimal result as,

Profit = \$ 233.16, EOQ (Q) = 16.82, S = 10.84, T = 0.6473 year, t_1= 0.5068 year, Deterioration cost = \$23.92, Shortage cost = \$24.81, Opportunity cost = \$0.93, Material cost = \$23.35, Holding cost = \$-119.97.

By comparing the above two examples, we get the lesser profit when the backlog rate is $\exp[-\delta(T-t)]$ under partial backlogging. All cost components except opportunity cost are low when the backlog rate is $1/(1+\delta(T-t)$. The convergence plots of the examples obtained using BCGA displayed in Figure 2 and 3 for backlog rates $B(T-t) = 1/(1+\delta(T-t))$ and $B(T-t) = \exp[-\delta(T-t)]$ respectively. We clearly see that, for reaching to the optimal profit of the formulated inventory model, the algorithm requires smaller number of generations. Hence, the algorithm has smaller computational complexity.

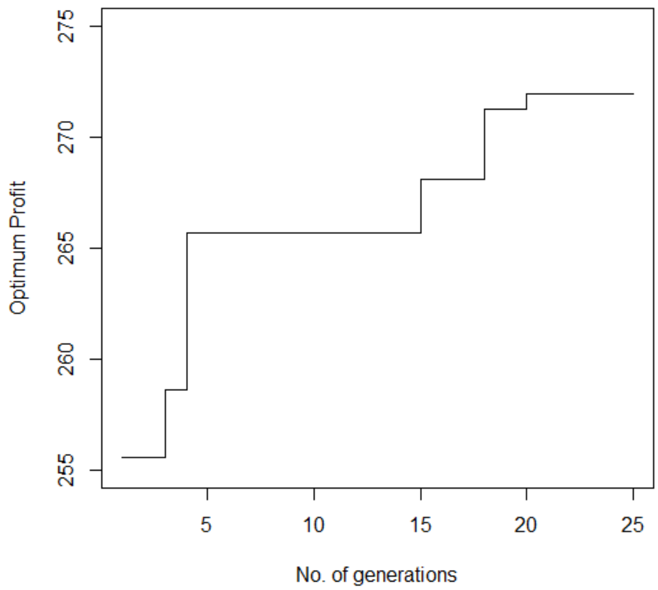

Fig. 2. Convergence plot of BCGA for backlog rate

$B(T-t) = 1/(1+\delta(T-t))$

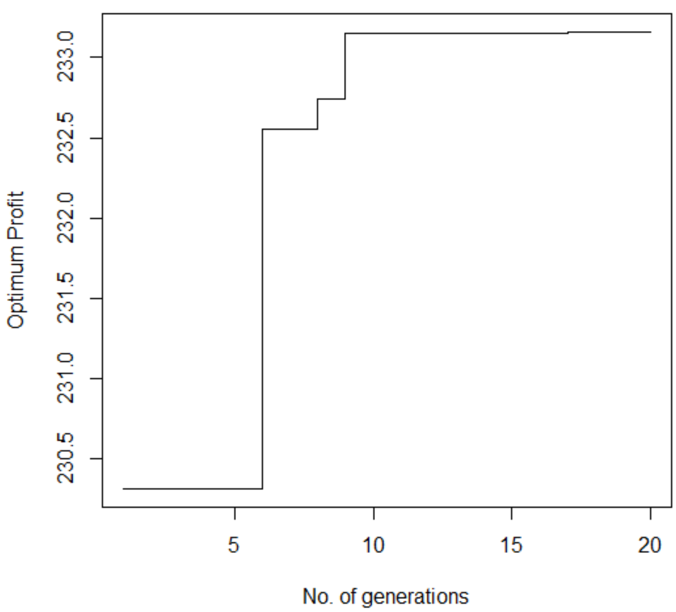

Fig. 3. Convergence plot of BCGA for backlog rate

$B(T-t) = \exp[-\delta(T-t)]$

VI. SENSITIVITY ANALYSIS

The results of sensitivity analysis are listed in Table I and II. The effect of a parameter is checked on optimum profit and EOQ by fixing other parameters. In Table I, the effect of various parameters viz., deterioration parameter, backlog parameter, holding cost parameters, unit cost, selling price, and shortage cost on profit function is checked for backlog rate,
$B(T-t) = 1/(1+\delta(T-t))$ and in Table II, for
$B(T-t) = \exp[-\delta(T-t)]$.

Table I. Effect of parameters on profit and EOQ for backlog rate $B(T-t) = 1/(1+\delta(T-t))$

Parameter	Value	Profit	EOQ
θ	0.5	1429.70	25.69
	0.7	467.43	18.35
	1.0	112.71	19.09
δ	1	295.00	19.57
	5	251.50	15.95
	7	244.15	15.59
α	5	781.84	34.41
	10	275.98	17.35
	15	140.19	14.31
β	1.5	155.06	17.25
	2.5	455.14	18.67
	5	2220.87	34.46
C	5	227.16	24.82
	7	240.58	19.82
	15	360.81	12.81
P	15	85.24	19.06
	20	182.34	17.88
	30	365.72	17.04
C_2	5	278.51	17.98
	10	275.04	17.11
	15	271.87	16.77

From Table I, following inferences are drawn:

- The smaller value of θ, α, δ and C_2 implies maximum profit and EOQ.
- When the deterioration parameter θ increases, the profit and EOQ of the system is decreased. Similar results are seen for α, δ and C_2.

Table II. Effect of parameters on profit and EOQ for backlog rate $B(T-t) = exp[-\delta(T-t)]$

Parameter	Value	Profit	EOQ
θ	0.5	1430.13	20.36
	0.7	464.67	15.21
	1.0	53.58	18.51
δ	1	238.65	18.65
	5	224.79	15.67
	7	223.54	15.39
α	5	780.74	33.69
	10	232.87	16.51
	15	80.08	16.25
β	1.5	96.07	17.73
	2.5	449.33	17.61
	5	2096.70	33.45
C	5	185.20	25.52
	7	196.69	22.53
	15	333.47	11.45
P	15	34.76	19.20
	20	135.46	18.11
	30	326.41	16.47
C_2	5	252.33	16.77
	10	228.51	17.33
	15	216.43	14.84

- For large value of β, C and P produce maximum profit. But for small values of C and P implies maximum EOQ and for large β gives maximum EOQ.
- From the results, optimum profit and EOQ are highly sensitive to changes in the value of deterioration parameter θ, selling price P and holding cost parameters α and β.
- Profit and EOQ are moderately sensitive to changes in the value of backlog parameter δ, purchase price C and shortage cost C_2.

From Table II, following inferences are drawn:

- The smaller value of θ, δ, α and C_2 implies maximum profit and EOQ.
- When the deterioration parameter θ increases, the profit and EOQ of the system is decreased. Similar results are seen for α, C and C_2.
- For large value of β and P produce maximum profit. But for small δ and P produce maximum EOQ and for large β gives maximum EOQ.
- The optimum profit and EOQ are highly sensitive to changes in the value of deterioration parameter θ, backlog parameter δ, selling price P and holding cost parameters α and β.
- Profit and EOQ are moderately sensitive to changes in the value of purchase price C and shortage cost C_2.

CONCLUSION

In this paper, an EOQ inventory model is developed. Various realistic assumptions such as price sensitive market demand rate, exponential distributed deterioration rate and linear time dependent holding cost are assumed for the development of the inventory system. Shortages are allowed with two different backlog rates are considered, to identify the most profitable and economical backlog rate. From the results we observed that, the backlog rate $1/(1+\delta(T-t))$ gave the maximum profit for the inventory system under partial backlogging. From the sensitivity analysis, deterioration parameter, backlog parameter, and holding cost parameters are the most sensitive on profit and EOQ. In future the formulated model can be extended for multi item inventory model with several price breaks, different deterioration rates such as three-parameter Weibull and Gamma, variable lead time etc. Also, the developed model can be solved by different mathematical and biological algorithms like Stochastic Annealing (SA).

APPENDIX

$$I(t) = Se^{-\theta t} - D(P)e^{-\theta t}\int_0^t e^{\theta x}dx \quad ; \quad 0 \le t \le t_1 \qquad (A.1)$$

$$I(t) = -D(P)\int_{t_1}^t \exp[\delta(T-t)]\ dx ; \quad t_1 \le t \le T \qquad (A.2)$$

Thus, the economic order quantity Q becomes,

$$Q = S + \int_{t_1}^T D(P)\exp[-\delta(T-x)]\,dx$$

$$= (a-bP)\left[\frac{(e^{\theta t_1}-1)}{\theta} + \int_{t_1}^T D(P)\exp[-\delta(T-x)]\,dx\right] \quad (A.3)$$

$$\text{Sales Revenue } = (a-Pb)P\left[t_1 + \int_{t_1}^T \exp[-\delta(T-x)]\,dx\right] \quad (A.4)$$

amount of shortage

$$= -\int_{t_1}^T\left[\int_{t_1}^t -D(P)\exp[-\delta(T-x)]\ dx\right]dt$$

$$= D(P)\int_{t_1}^T\left[\int_{t_1}^t \exp[-\delta(T-x)]\ dx\right]dt$$

$$\text{Shortage cost} = C_2 * \text{amount of shortage}$$

$$= C_2(a-Pb)\int_{t_1}^T\left[\int_{t_1}^t \exp[-\delta(T-x)]\ dx\right]dt \qquad (A.5)$$

The amount of lost sales during $[t_1, T]$ is,

$$\text{amount of lost sales } = D(P)\int_{t_1}^T[1-\exp[-\delta(T-x)]]\,dt$$

$$= (a-Pb)\int_{t_1}^T[1-\exp[-\delta(T-x)]]\,dt$$

The opportunity cost during $[t_1, T]$ is,

$$\text{opportunity cost} = C_3 * \text{amount of lost sales}$$

$$= C_3(a-Pb)\int_{t_1}^T[1-\exp[-\delta(T-x)]]\,dt \qquad (A.6)$$

$$\text{Material cost} = C\left[\int_0^{t_1} I(t)\,dt - \int_{t_1}^T I(t)\,dt\right]$$

$$= (a-Pb)C\left[\begin{array}{c}\frac{(e^{-\theta t_1}-1)(1-e^{\theta t_1})}{\theta} \\ -\frac{\theta t_1 + (e^{-\theta t_1}-1)}{\theta^2} \\ + \int_{t_1}^T\left[\int_{t_1}^t \exp[-\delta(T-x)]\ dx\right]\ dt\end{array}\right] \qquad (A.7)$$

ACKNOWLEDGMENT

The author is grateful to the editor and anonymous reviewers of the national conference on “Trends and Perspectives in Statistics and Mathematics: A multi-Disciplinary Approach” for their critical meticulous comments and suggestions which helped the author to improve the manuscript in the present stage.

REFERENCES

Agi, M.A. & Soni, H.N. (2020). Joint pricing and inventory decisions for perishable products with age-, stock-, and price-dependent demand rate. *Journal of the Operational Research Society*, 71(1), 85-99.

Alfares, H.K. & Ghaithan, A.M. (2016). Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. *Computers & Industrial Engineering*, 94, 170-177.

Begum, R., Sahoo, R.R. & Sahu, S.K. (2012). A replenishment policy for items with price dependent demand, time-proportional deterioration and no shortages. *International Journal of Systems Science*, 43(5), 903-910.

De, S.K. & Sana, S.S. (2015). Backlogging EOQ model for promotional effort and selling price sensitive demand-an intuitionistic fuzzy approach. *Annals of Operations Research*, 233(1), 57-76.

Deb K. (2001). Multi-Objective Optimization Using Evolutionary Algorithm. *John Wiley & Sons*.

Dye, C.Y., Hsieh, T.P. & Ouyang, L.Y. (2007). Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. *European Journal of Operational Research*, 181(2), 668-678.

Goh, M. (1994). EOQ models with general demand and holding cost functions. *European Journal of Operational Research*, 73(1), 50-54.

Khanra, S., Sana, S.S. & Chaudhuri, K. (2010). An EOQ model for perishable item with stock and price dependent demand rate. *International Journal of Mathematics in Operational Research*, 2(3), 320-335.

Kurade, S. S. & Latpate, R. (2020). Demand and deterioration of items per unit time inventory models with shortages using genetic algorithm. *Journal of Management Analytics*, 8(3), 502-529. DOI: 10.1080/23270012.2020.1829113.

Latpate, R. & Kurade, S.S. (2020). Multi-objective multi-index transportation model for crude oil using fuzzy NSGA-II. *IEEE Transactions on Intelligent Transportation Systems*. DOI: 10.1109/TITS.2020.3024693.

Latpate, R. & Kurade, S. (2017). Fuzzy MOGA for supply chain models with Pareto decision space at different α-cuts. *International Journal of Advanced Manufacturing Technology*, 91, 3861-3876.

Li, R. & Teng, J.T. (2018). Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. *European Journal of Operational Research*, 270(3), 1099-1108.

Mondal, B., Bhunia, A.K. & Maiti, M. (2003). An inventory system of ameliorating items for price dependent demand rate. *Computers & Industrial Engineering*, 45(3), 443-456.

Mukhopadhyay, S., Mukherjee, R.N. & Chaudhuri, K.S. (2005). An EOQ model with two-parameter Weibull distribution deterioration and price-dependent demand. *International Journal of Mathematical Education in Science and Technology*, 36(1), 25-33.

Panda, S., Saha, S. & Basu, M. (2009). An EOQ model for perishable products with discounted selling price and stock dependent demand. *Central European Journal of Operations Research*, 17(1), 31-53. DOI: 10.1007/s10100-008-0073-z.

Ruidas, S., Seikh, M.R. & Nayak, P.K. (2020). An EPQ model with stock and selling price dependent demand and variable production rate in interval environment. *International Journal of System Assurance Engineering and Management*, 11, 385-399. DOI: 10.1007/s13198-019-00867-w.

Sarkar, B., Saren, S. & Wee, H.M. (2013). An inventory model with variable demand, component cost and selling price for deteriorating items. *Economic Modelling*, 30, 306-310.

Sana, S. S. (2010). Optimal selling price and lotsize with time varying deterioration and partial backlogging. *Applied Mathematics and computation*, 217(1), 185-194.

Wee, H. M. (1997). A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. *Production Planning and Control*, 8(5), 494-499.

Wu, J., Chang, C.T., Teng, J.T. & Lai, K.K. (2017). Optimal order quantity and selling price over a product life cycle with deterioration rate linked to expiration date. *International Journal of Production Economics*, 193, 343-351.

Yang, C.T. (2014). An inventory model with both stock-dependent demand rate and stock-dependent holding cost rate. *International Journal of Production Economics*, 155, 214-221.

***