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Changing and Unchanging the Signal Number: Edge Removal

[1]R. Antony Doss and [2] S. Balamurugan

[1]Department of Mathematics, Affiliated To Manonmaniam Sundaranar University, Tirunelveli, INDIA.
E-mail: antonydoss2193@gmail.com
[2]St. Xavier's College (Autonomous), Palayamkottai, E-mail: balamaths@rocketmail.com

Abstract -: For two vertices u and v in a connected graph G, the signal distance d_{SD} (u, v) from u to v is defined by d_{SD} (u, v) = $\min_S\{d(u,v) + \sum_{w\in V(G)} (\deg w - 2) + (\deg u - 1) + (\deg v - 1)\}$, where S is a path connecting u and v, $d(u, v)$ is the length of the path S and in the sum $\sum_{w\in V(G)}$ runs over all the internal vertices between u and v in the path S. A path between the vertices u and v of length d_{SD} (u, v) is called a $u - v$ geosig path. A set $S \subseteq V$ is called a signal set, if every vertex x in G lies on a geosig path joining a pair of vertices of S. This paper, we study about the criticality and stability of the signal number after removing an edge from a graph.

Keywords: Signal distance, signal set, signal number.

I. INTRODUCTION

Initially, a new distance parameter was introduced by K. M. Kathiresan which is called as the signal distance in graphs, refer [3]. After there was an extension of this signal distance in graphs done by S. Sethu Ramalingam and S. Balamurugan and came with the signal number, refer [4]. Further, new signal numbers such as the independent signal number and the irredundant signal number, were introduced and studied by S. Balamurugan and R. Antony Doss, refer [1]. Now, in this paper, we are going to study the criticality and stability of the signal number which is about removing an edge from a graph.

The concept of changing and unchanging the signal number is new to this parameter. In many areas, changing and unchanging of the signal number focusing on what kind of impact happen when an edge is removed. In this paper, we investigate that, the signal number $sn(G)$ of a graph G either increase or decrease or unchanged when an edge is removed from G. Symbols and notations used here are not mentioned, refer [2]. Throughout this paper we mention an edge as **$e = (u, v) \in E(G)$** also we denote the geosig path between any two vertices as **gs path**.

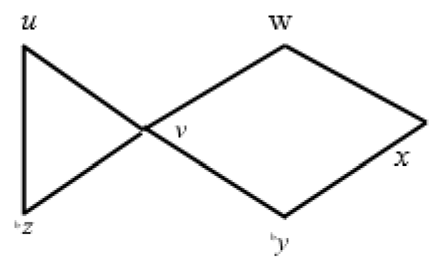

Figure.1 Graph G

Example 1.1.In the above Figure, the set is $\{u, x, z\}$ forms a signal set and any set with two vertices will not form a signal set, we have $sn(G) = 3$. *Upon the removal of every edge of G the signal sets and the signal numbers are given in the following table.*

Edge (e)	Signal set of $(G - e)$	$sn(G - e)$
(u, z)	$\{u, x, z\}$	3
(u, v)	$\{u, x\}$	2
(v, w)	$\{u, w, x, z\}$	4
(w, x)	$\{u, w, z\}$	3
(x, y)	$\{u, x, y, z\}$	4
(y, z)	$\{u, y, z\}$	3
(v, z)	$\{x, z\}$	2

The theorems mention below are needed in the following sections:

Theorem 1.2. [1] For any graph G of order $n(\geq 3)$, $sn(G) = n$ *if and only if* $G = K_n$.

Theorem 1.3. [1] Let C_n *be a cycle on n vertices. Then*

(i) $sn_{ir}(C_n) = sn(C_n) = sn_i(C_n) = \begin{cases} 2, & n \text{ is even} \\ 3, & n \text{ is odd} \end{cases}$

(ii) $sn_i^+(C_n) = sn^+(C_n) = sn_{ir}^+(C_n) = 3.$

II. 2. Common Classes of Graphs

This section, we examine the criticality and stability of paths, cycles and other graphs.

Theorem 2.1. *For any* P_n $(n > 2)$, $sn(P) = 4$, where $P = P_n - e$ and $e \in E(P_n)$.

Proof. Suppose we assume $P_n = (v_1, ..., v_n)$ is a path. Clearly, the set $\{v_1, v_n\}$ is a signal set of P_n, where v_1 and v_n are pendent vertices. Suppose e is not incident with the pendent vertex.

Then, we have two paths P_1 and P_2 of cardinality strictly less than n. Clearly, the signal sets of P_1 and P_2 must only have pendent vetices and so we have $sn(P_1) = 2$ and $sn(P_2) = 2$. Therefore $sn(P) = sn(P_1) + sn(P_2) = 2 + 2 = 4$. Hence we proved the theorem.

Remark 2.2. Now, if we remove the edge which is incident with the end vertex of P_n, then we have two paths P_1 and P_{n-1}. Now, for P_1 we cannot able to define a signal number and for P_{n-1}, the signal set has two pendent vertices therefore again $sn(P_{n-1}) = 2$. But, we cannot define a signal number for this case.

Conjecture 2.3. For any path P_n $(n > 2)$ and $P = P_n - e$, $sn_{ir}(P) = sn(P) = sn_i(P) = sn_i^+(P) = sn^+(P) = sn_{ir}^+(P) = 4$, where $e \in E(P_n)$.

Theorem 2.4. Let K_n $(n > 1)$ be a complete graph and $K = K_n - e$, where $e \in E(K_n)$. *Then* $sn(K) = 2$.

Proof. From the Theorem 1.2, we have $sn(K_n) = n$. Suppose we remove an edge $e \in E(G)$. Then we get a new graph, which is, $K_{2,n}$ with $2, n$ vertices. Now, we have two partitions $|U| = \{u_1, u_2\}$ and $|V| = \{v_1, ..., v_n\}$. It is clear that, all the n vertices are covered by the vertices u_1 and u_2. Therefore, the signal set has two vertices. Hence we proved $sn(K) = 2$.

Conjecture 2.5. For $K = K_n - e$, *where* $e \in E(K_n)$, $sn_{ir}(K) = sn(K) = sn^+(K) = sn_{ir}^+(K) = 2$

Theorem 2.6. For a cycle C_n $(n \geq 3)$, $sn(C) = 2$, *for all n, where* $C = C_n - e$ *and* $e \in E(C_n)$.

Proof. From Theorem 1.3, we have $sn(C_n) = 2$, n is even and $sn(C_n) = 3$, n is odd. Suppose we remove e, then, the resulting graph is a path on n vertices, for all n. Now the graph C has two pendent vertices, v_1, v_n which cover all the remaining vertices. Therefore $sn(C) = 2$, for all n.

Theorem 2.7. Let C_n $(n \geq 3)$ be a cycle on n vertices and $C = C_n - e$ *and* $e \in E(C_n)$. *Then* $sn_{ir}(C) = sn(C) = sn_i(C) = sn_i^+(C) = sn^+(C) = sn_{ir}^+(C) = 2$.

Proof: By the above Theorem, we proved sn (C) = 2, where $C_n - e\ E(C_n)$ *also we know that,* $C_n - e = P_n$. *Clearly, C has two end vertices, say* v_1, v_n.. *Since we proved for a path* P_n *on vertices,* $sn_{ir}(P_n) = sn(P_n) = sn_i(P_n) = sn_i^+(P_n) = sn^+(P_n) = sn_{ir}^+(P_n) = 2$. Therefore, we get $sn_{ir}(C) = sn(C) = sn_i(C) = sn_i^+(C) = sn^+(C) = sn_{ir}^+(C) = 2$. Hence we proved the theorem.

Theorem 2.8. For $K_{m,n}$ $(2 < m < n)$ *and* $e \in E(K_{m,n})$, $sn(K_1) = 2$, where $K_1 = K_{m,n} - e$.

Proof. Suppose we assume U and V are the two partitions with orders m and n of $K_{m,n}$. Now, the resulting graph after the removal of e is again $K_{m,n}$. Now, we assume a vertex of U and a vertex of V form a signal set of $K_{m,n}$. Obviously, the other vertices are covered by u and v. Then (u, v_s, u_r, v), for $r = 1, 2, ..., n$ and $s = 1, 2, ..., m$ is the gs path between u and v with $d_{SD}(u, v) = 2m + 2n - 2$, which is true for any $u_r, v_s \in V(K_{m,n})$. Also, it is easy to see, the $d_{SD}(u, v)$ of other paths between u and v, with more than two internal vertices, are strictly greater than $2m + 2n - 2$. Therefore (u, v_s, u_r, v) is the gs path between u and v. *Therefore,* $\{u, v\}$ is a sn-set of cardinality 2. Hence we proved $sn(K_1) = 2$.

Theorem 2.9. Let $K_{m,n}$, $1 < m \leq n$ *be a complete bipartite graph and* $K_1 = K_{m,n} - e$. *where* $e \in E(K_{m,n})$. Then we have the following,

(i) $sn_{ir}(K_1) = sn(K_1) = sn_i(K_1) = 2$

(ii) $sn_i^+(K_1) = sn^+(K_1) = sn_{ir}^+(K_1) = n$.

Proof: i). *Suppose we assume U and V are the two partitions with orders m and n of* $K_{m,n}$. By the above Theorem, we proved $sn(K) = 2$, where $K_1 = K_{m,n} - e$. *where* $e \in E(K_{m,n})$. Now, we have only to prove $sn_{ir}(K_1) = sn_i(K_1) = 2$. Suppose we assume S = {u, v}, where $u \in U$ and $v \in V$ as a signal set of K_1. Then we have to prove $|S| = sn_{ir}(K_1) = sn_i(K_1)$. It is easy to see and also by the definition of sn_{ir}-set, S is also an sn_{ir}-set. Therefore, $|S| = sn_{ir}(K_1)$. Now, we are remaining to prove $|S| = sn_i(K_1)$. Suppose, we assume $sn(K) < sn_i(K_1)$. Then the signal set S_1 has more than one vertex than S. Now, $S_1 = \{u, x, v\}$, for some x either in U or V. As a sn-set, gs paths of vertices of S_1 must cover the other vertices of K_1. But the vertex x is covered by u – v gs path. Therefore, we get a contradiction to our assumption that $x \in S_1$. Hence we proved $|S| = sn_i(K_1)$.

ii). It is clear that |V| is the maximum signal set of K_1. Also, vertices of V all are independent and if we remove any vertex from the sn^+-set, then that vertex is not covered by any of the other vertices of the same set. From this, V is basically a maximum sn_{ir}^+-set. Hence we proved $sn_i^+(K_1) = sn^+(K_1) = sn_{ir}^+(K_1) = n$.

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