Abstract: The study proposed an alternative imputation scheme for the schemes which converged to sample mean as the values of unknown parameters in their estimators converged to zero. The estimator of the population means for the proposed scheme as well as the bias and MSE were derived. The efficiency condition under which the modified estimator is more efficient than existing ones were also presented. Empirical study using four sets of populations was conducted and the results revealed that the proposed estimator was more efficient.

Index Terms: Estimator; Population mean; Imputation scheme; Bias; Mean Squared Error (MSE)

I. INTRODUCTION

Data obtained from sampling surveys often face the problem of non-response or missing values. These missing values create difficulty in analysis, processing and handling of data. The problem of non-response has been considered by many authors including Singh and Horn (2000), Singh and Deo (2003), Wang and Wang (2006), Kadilar and Cingi (2008), Toutenburg et al. (2008), Singh (2009), Diana and Perri (2010), Al-Omari et al. (2013), Singh et al. (2014), Gira (2015), Singh et al. (2016), Singh, et al. (2010), Bhushan and Pandey (2016) and Prasad (2017), Audu et al. (2020a, b, c), Audu et al. (2021), Singh and Deo (2003) and Prasad (2017) estimators converged to sample mean as the values of unknown parameters in their estimators converged to zero while Singh and Horn (2000), Singh et al. (2014) estimators converged to sample mean as the values of unknown parameters converged to one. These converges lead to lose of information on the auxiliary variables which in turn reduces their efficiencies. These limitations identified above prompt the present study, some existing literature on imputation schemes and estimators are reviewed and their properties were presented.

II. METHODOLOGY

A. Some Existing Related Imputation Schemes

Let $R$ denotes the set of $r$ unit’s response and $R^c$ denotes the set of $n-r$ unit’s non-response or missing out of $n$ units sampled without replacement from the $N$ unit’s population. For each $i \in R$, the value of $y_i$ is observed. However, for unit $i \in R^c$, $y_i$ is missing but calculated using different methods of imputation.

The Mean Method of imputation, values found missing are to be replaced by the mean of the rest of observed values. The study variable thereafter, takes the form given as,

$$
y_j = \begin{cases} 
  y_i & \text{if } i \in R \\
  y_r & \text{if } i \in R^c
\end{cases}
$$

(1)

Under the method of imputation, sample mean denoted by $\hat{\theta}_{mean}$ can be derived as

$$
\overline{y}_r = \frac{1}{r} \sum_{i \in R} y_i
$$

(2)
The bias and MSE of \( \hat{\theta}_{\text{mean}} \) are given by (3) and (4) respectively.

\[
\text{Bias}(\hat{\theta}_{\text{mean}}) = 0
\]  

\[
\text{MSE}(\hat{\theta}_{\text{mean}}) = \varphi_{r,N} \bar{Y}^2 C_Y^2
\]  

where

\[
\varphi_{r,N} = 1 + \frac{1}{r} - \frac{1}{N}, \quad C_r = \frac{S_y}{\bar{Y}}, \quad S_r = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{Y})^2, \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i
\]

Under ratio method of imputation, values found missing in the study variable is to be replaced by values obtained using the expression

\[
\hat{y}_j = \frac{\beta x_i}{r} \quad i \in R^c
\]

The study variable thereafter, takes the form given as

\[
y_j = \begin{cases} 
    y_i & i \in R \\
    \hat{y}_j & i \in R^c
  \end{cases}
\]  

The estimator of population mean denoted by \( \hat{\theta}_{\text{ratio}} \), its bias and MSE under the method of ratio imputation are given as

\[
\hat{\theta}_{\text{ratio}} = \frac{\bar{y}_r}{\bar{x}_r}
\]

\[
\text{Bias}(\hat{\theta}_{\text{ratio}}) = \varphi_{r,N} \bar{Y} \left( C_X^2 - C_{XY} \right)
\]

\[
\text{MSE}(\hat{\theta}_{\text{ratio}}) = \varphi_{r,N} \bar{Y}^2 \left( C_X^2 + \varphi_{r,N} \bar{Y}^2 \left( C_X^2 - 2C_{XY} \right) \right)
\]

where

\[
C_{XY} = \rho_{X,Y} C_X C_Y, \quad \rho_{X,Y} = \frac{S_{xy}}{S_x S_y}, \quad C_X = \frac{S_x}{\bar{X}}, \quad S_x = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{X})^2
\]

\[
S_{XY} = \frac{1}{N-1} \sum_{i=1}^{N} \left( y_i - \bar{Y} \right) \left( x_i - \bar{X} \right), \quad \bar{X} = \frac{1}{N} \sum_{i=1}^{N} x_i, \quad \varphi_{r,n} = 1 - \frac{1}{n}
\]

Singh and Horn (2000) utilized information from imputed values for responding and non-responding units as well, thereafter giving study variable the form given by (9).

\[
y_j = \begin{cases} 
    \frac{\lambda}{r} y_i + \left( 1 - \lambda \right) \beta x_i & i \in R \\
    \left( 1 - \lambda \right) \beta x_i & i \in R^c
  \end{cases}
\]

Under this method of imputation, estimator of population mean denoted by \( \hat{\theta}_{\text{comp}} \) can be derived as

\[
\hat{\theta}_{\text{comp}} = \bar{y}_r \left( \lambda + \left( 1 - \lambda \right) \frac{\bar{X}}{\bar{x}_n} \right)
\]

The bias and MSE of \( \hat{\theta}_{\text{comp}} \) up to first order approximation are given by (11) and (12) respectively as:

\[
\text{Bias}(\hat{\theta}_{\text{comp}}) = (1 - \lambda) \varphi_{r,n} \bar{Y} \left( C_X^2 - C_{XY} \right)
\]

\[
\text{MSE}(\hat{\theta}_{\text{comp}}) = \varphi_{r,n} \bar{Y}^2 C_Y^2 + \varphi_{r,n} \bar{Y}^2 \left( (1 - \lambda)^2 C_X^2 - 2(1 - \lambda) C_{XY} \right)
\]

\[
\hat{\theta}_{\text{comp}} \text{ attained optimality when } \lambda = 1 - C_{XY} / C_X^2 \text{ and the minimum MSE of } \theta_{\text{comp}} \text{ denoted by }
\]

\[
\text{MSE}(\hat{\theta}_{\text{comp}})_{\text{min}} \text{ is given by }
\]

\[
\text{MSE}(\hat{\theta}_{\text{comp}})_{\text{min}} = \bar{Y}^2 C_Y^2 \left( \varphi_{r,N} - \varphi_{r,n} \rho_{XY}^2 \right)
\]

Singh and Deo (2003) incorporated power transformation parameter to \( \hat{\theta}_{\text{ratio}} \) and obtain \( \hat{\theta}_{\text{SD}} \) as

\[
\hat{\theta}_{\text{SD}} = \bar{y}_r \left( \frac{\bar{X}_n}{\bar{x}_r} \right)^{\alpha}
\]

\[
\hat{\theta}_{\text{SD}} \text{ attained optimum when } \alpha = \frac{R S_{XY}}{S_X^2} \text{ and the MSE(\hat{\theta}_{\text{SD}})}_{\text{min}} \text{ is given by }
\]

\[
\text{MSE(\hat{\theta}_{\text{SD}})}_{\text{min}} = \text{MSE(\hat{\theta}_{\text{ratio}})}_{\text{min}} \left( \beta_{\text{reg}} - R \right)^2
\]

where \( \beta_{\text{reg}} = S_{XY} / S_X^2 \).

Kadilar and Cingi (2008) modified the work of Kadilar and Cingi (2004) in the case of missing observations and suggested the following estimators of population mean

\[
\hat{\theta}_{KC1} = \left( \bar{y}_r + \hat{\beta}_{\text{reg}} \left( \bar{X} - \bar{x}_n \right) \right) \frac{\bar{X}}{\bar{x}_r}
\]

\[
\hat{\theta}_{KC2} = \left( \bar{y}_r + \hat{\beta}_{\text{reg}} \left( \bar{X} - \bar{x}_n \right) \right) \frac{\bar{X}}{\bar{x}_r}
\]

\[
\hat{\theta}_{KC3} = \left( \bar{y}_r + \hat{\beta}_{\text{reg}} \left( \bar{X} - \bar{x}_n \right) \right) \frac{\bar{X}}{\bar{x}_r}
\]
The MSEs of $\hat{\theta}_{KC1}, \hat{\theta}_{KC2}$ and $\hat{\theta}_{KC3}$ are given in (19), (20) and (21) respectively

$$MSE(\hat{\theta}_{KC1}) = MSE(\hat{\theta}_{mean}) + \psi_{r,N} S_X^2 (R^2 - \beta_{reg}^2)$$ (19)

$$MSE(\hat{\theta}_{KC2}) = MSE(\hat{\theta}_{mean}) + \psi_{n,N} S_X^2 (R^2 - \beta_{reg}^2)$$ (20)

$$MSE(\hat{\theta}_{KC3}) = MSE(\hat{\theta}_{mean}) + \psi_{r,n} S_X^2 (R + \beta_{reg})^2 - 2(R + \beta_{reg}) S_{XY}$$ (21)

Singh et al. (2014) proposed Exponential-Type Compromised Imputation method as

$$\hat{y}_i = \begin{cases} n/r \times y_i + (1-n/r) \bar{y}_r \exp \left( \frac{\bar{x} - x_i}{\bar{x} + x_i} \right) & \text{if } i \in R \\ (1-n/r) \times \bar{y}_r \exp \left( \frac{\bar{x} - x_i}{\bar{x} + x_i} \right) & \text{if } i \in R^c \end{cases}$$ (22)

The point estimator $\hat{\theta}_{ExpCmp}$, bias and MSE are given as:

$$\hat{\theta}_{ExpCmp} = n/r \times \bar{y}_r + (1-n/r) \bar{y}_r \exp \left( \frac{\bar{x} - x}{\bar{x} + x} \right)$$ (24)

$$Bias(\hat{\theta}_{ExpCmp}) = (1-n/r) \times \phi_{r,N} \bar{y} \left( \frac{3}{8} C_X^2 - \frac{1}{2} C_{XY} \right)$$ (25)

$$MSE(\hat{\theta}_{ExpCmp}) = \phi_{r,N} \bar{y}^2 \left( C_X^2 + \frac{(1-n/r)^2}{4} - (1-n/r)C_{XY} \right)$$ (26)

$$MSE(\hat{\theta}_{ExpCmp})_{min} = \phi_{r,n} \bar{y}^2 C_X^2 \left( 1 - \rho_{XY}^2 \right)$$ (27)

Prasad (2017) proposed ratio exponential estimator given as

$$\hat{\theta}_{Prasad} = \eta \bar{y}_r \exp \left( \frac{\bar{x} - x_i}{\bar{x} + x_i + 2 \rho_{XY} / \beta_2(x)} \right)$$ (28)

where

$$\eta = \bar{y}^2 / \left( \bar{y}^2 + \phi_{r,N} \left( S_Y^2 + 0.25 \mu_X^2 R^2 S_X^2 - \mu_R S_{XY} \right) \right)$$

$$MSE(\hat{\theta}_{Prasad})_{min} = \bar{y}^2 \left( \phi_{r,N} \left( C_Y^2 + 0.25 \mu_X^2 C_X^2 - \mu C_{XY} \right) \right)$$ (29)

### III. PROPOSED MODIFIED SCHEME

Motivated by the schemes proposed by Singh and Horn (2000), Singh and Deo (2003), Singh et al. (2014) and Prasad (2017), the following imputation scheme for estimating finite population mean is suggested;

$$\bar{y}_r = \frac{1}{n-r} \left( n \left( \theta \bar{y}_r + (1-\theta) \bar{y} \right) \frac{x_i}{\bar{x}} \exp \left( \frac{x_i - \bar{x}}{\bar{x} + x_i} - r \right) \right)$$ if $i \in R$ (30)

where $\theta_i$ is unknown function of study and auxiliary variables which minimized the mean squared error of the proposed scheme.

A. Equation of Proposed Estimator of the Modified Scheme

The point estimator for the proposed scheme is obtained as

$$i_{new} = \left( \theta \bar{y}_r + (1-\theta) \bar{y} \right) \frac{x_i}{\bar{x}} \exp \left( \frac{x_i - \bar{x}}{\bar{x} + x_i} \right)$$ (31)

Simplify (31) to obtain the estimator of the modified scheme

$$i_{new} = \left( \theta \bar{y}_r + (1-\theta) \bar{y} \right) \exp \left( \frac{\bar{x} - x_i}{\bar{x} + x_i} \right)$$ (32)

B. Properties (Bias and MSE) for Estimator of the Modified Scheme

In this subsection, the bias and MSE for estimator of the modified scheme are derived and discussed.

To derive the properties of the estimator obtained from the modified scheme, the following error terms are defined:

$$e_0 = \frac{\bar{y}_r - \bar{y}}{\bar{y}}, \quad e_1 = \frac{\bar{x} - \bar{y}}{\bar{y}} \quad \text{such that } r \to n, \quad \lim |e_i| = 0, i = 0, 1$$

This implies that $\bar{y}_r = \bar{y} \left( 1 + e_0 \right), \quad \bar{x} = \bar{y} \left( 1 + e_1 \right)$.

The expectation in terms of $e_0$ and $e_1$ are given in (33);

$$E(e_0) = E(e_1) = 0, \quad E(e_0^2) = \left( 1 - \frac{1}{N} \right) C_Y^2$$

$$E(e_1^2) = \left( 1 - \frac{1}{N} \right) C_X^2, \quad E(e_0 e_1) = \left( 1 - \frac{1}{N} \right) \rho_{XY} C_Y C_X$$ (33)

Similarly, express (32) in terms of error terms $e_0$ and $e_1, i_{new}$ is of the form given in (34)

$$i_{new} = \left( \theta \bar{y} \left( 1 + e_0 \right) + (1-\theta) \bar{y} \left( 1 + e_1 \right) \right) \frac{x_i}{\bar{x}} \exp \left( \frac{\bar{x} - \bar{x} \left( 1 + e_1 \right)}{\bar{x} + x_i} \right)$$ (34)
Simplify (34) up to first order approximation,
\[ t^{\text{new}} = \bar{Y} \left( 1 + e_0 + \left( \frac{1}{2} - \theta_2 \right) e_1 - \left( \frac{1}{8} - \theta_2 \right) e_1^2 - \left( \frac{1}{2} + \theta_2 \right) e_0 e_1 \right) \] (35)

Subtract \( \bar{Y} \) from both sides of (35)
\[ t^{\text{new}} - \bar{Y} = \bar{Y} \left( e_0 + \left( \frac{1}{2} - \theta_2 \right) e_1 - \left( \frac{1}{8} - \theta_2 \right) e_1^2 - \left( \frac{1}{2} + \theta_2 \right) e_0 e_1 \right) \] (36)

Take expectation of (36) and apply the results of (33), the bias of \( t_{2}^{\text{new}} \) is obtained as:
\[ \text{Bias}(t_{2}^{\text{new}}) = \bar{Y} \left( \frac{1}{r} - \frac{1}{N} \right) \left( \frac{1}{8} - \theta_2 \right) \rho_{yx} C_{x} C_{y} \] (37)

Similarly, square both sides of (36) up to second degree approximation, take expectation and apply the results of (33), the MSE of \( t_{2}^{\text{new}} \) is obtained as:
\[ \text{MSE}(t_{2}^{\text{new}}) = \bar{Y}^2 \left( \frac{1}{r} - \frac{1}{N} \right) \left( \frac{1}{2} - \theta_2 \right)^2 C_{x}^2 + 2 \left( \frac{1}{2} - \theta_2 \right) \rho_{yx} C_{y} C_{x} \] (38)

C. Minimum MSEs of \( t_{2}^{\text{new}} \)

To obtain the minimum MSE of \( t_{2}^{\text{new}} \), (38) is partially differentiated with respect to \( \theta_2 \) respectively and equate to zero as:
\[ \frac{\partial \text{MSE}(t_{2}^{\text{new}})}{\partial \theta_2} = \bar{Y}^2 \left( \frac{1}{r} - \frac{1}{N} \right) \left( -2 \left( \frac{1}{2} - \theta_2 \right) C_{x}^2 - 2 \rho_{yx} C_{y} C_{x} \right) = 0 \] (39)

Solve for \( \theta_2 \) in (39), we obtained
\[ \theta_2 = \frac{1}{2} + \frac{\rho_{yx}}{C_y} \] (40)

Substitute (40) in (39) respectively, the minimum MSE of \( t_{2}^{\text{new}} \) is obtained as
\[ \text{MSE}(t_{2}^{\text{new}}) = \bar{Y}^2 \left( \frac{1}{r} - \frac{1}{N} \right) C_{y}^2 \left( 1 - \rho_{yx}^2 \right) \] (41)

D. Efficiency Comparison of Proposed Estimator \( t_{2}^{\text{new}} \) with some Related Estimators

i. \( \text{MSE}(\hat{\theta}_{\text{mean}}) - \text{MSE}(t_{2}^{\text{new}}) > 0 \)
(46) $MSE(\hat{\theta}_{k1}) - MSE(\hat{\theta}_{k2}) = 0$

\[
(1 - \frac{1}{N}) y^2 c^2 + \psi_{xy} s_{xy}^2 (R - B_{xy}) - \frac{2}{N} \psi_{xy} s_{xy}^2 (1 - \frac{1}{N}) \frac{1}{2 - \theta_1} c^2 = 0
\]

\[
\theta_1 < \frac{1}{2} - \frac{\rho_{xy}}{C_x} + \frac{1}{C_x} \left[ \psi_{xy} s_{xy}^2 (R - B_{xy}) \left( \frac{1}{2} - \frac{1}{N} \right)^{-1} \right] - \rho_{xy} C_x > 0
\]

(47) $MSE(\hat{\theta}_{k2}) - MSE(\hat{\theta}_{k3}) = 0$

\[
\psi_{xy} s_{xy}^2 (R - B_{xy}) - \frac{2}{N} \psi_{xy} s_{xy}^2 \left( \frac{1}{2} - \frac{1}{N} \right)^{-1} - \rho_{xy} C_x > 0
\]

\[
\psi_{xy} s_{xy}^2 (R - B_{xy}) - \frac{2}{N} \psi_{xy} s_{xy}^2 \left( \frac{1}{2} - \frac{1}{N} \right)^{-1} - \rho_{xy} C_x > 0
\]

\[
\psi_{xy} s_{xy}^2 (R - B_{xy}) - \frac{2}{N} \psi_{xy} s_{xy}^2 \left( \frac{1}{2} - \frac{1}{N} \right)^{-1} - \rho_{xy} C_x > 0
\]

IV. EMPIRICAL STUDY

In this section, efficiency of the proposed modified estimator was compared with that of some existing estimators numerically.

### Table 1: Data used for empirical study

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>80</td>
<td>80</td>
<td>10</td>
<td>284</td>
</tr>
<tr>
<td>$n$</td>
<td>25</td>
<td>25</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>$r$ (Assumed)</td>
<td>20</td>
<td>20</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>5182.638</td>
<td>5182.638</td>
<td>56.9</td>
<td>29.36</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>285.122</td>
<td>1126.463</td>
<td>54.2961</td>
<td>245.088</td>
</tr>
<tr>
<td>$C_x$</td>
<td>0.3542</td>
<td>0.3542</td>
<td>0.1840</td>
<td>1.76</td>
</tr>
<tr>
<td>$C_y$</td>
<td>0.9485</td>
<td>0.7507</td>
<td>0.1621</td>
<td>2.43</td>
</tr>
<tr>
<td>$\beta_1(x)$</td>
<td>1.2680</td>
<td>1.0237</td>
<td>0.4956</td>
<td>8.77</td>
</tr>
<tr>
<td>$\beta_2(x)$</td>
<td>3.5360</td>
<td>2.8306</td>
<td>2.5932</td>
<td>88.88</td>
</tr>
<tr>
<td>$\rho_{xy}$</td>
<td>0.9140</td>
<td>0.9140</td>
<td>0.9237</td>
<td>0.961</td>
</tr>
</tbody>
</table>

### Table 2: MSE of Some Estimators and $\hat{\theta}_{new}$ using Populations 1 & 2

<table>
<thead>
<tr>
<th>Estimators</th>
<th>Population 1</th>
<th>Population 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIAS</td>
<td>MSE</td>
</tr>
<tr>
<td>$\hat{\theta}<em>{wm, \hat{\theta}</em>{new}}$</td>
<td>0</td>
<td>126366</td>
</tr>
<tr>
<td>$\hat{\theta}_{mo}$</td>
<td>-96084.0</td>
<td>203055.8</td>
</tr>
<tr>
<td>$\hat{\theta}_{mb}$</td>
<td>89.97976</td>
<td>98215.14</td>
</tr>
<tr>
<td>$\hat{\theta}_{m}$</td>
<td>-10.6729</td>
<td>98215.14</td>
</tr>
<tr>
<td>$\hat{\theta}_{mmb}$</td>
<td>13.53135</td>
<td>20800.34</td>
</tr>
<tr>
<td>$\hat{\theta}_{mc}$</td>
<td>-310.565</td>
<td>926966.2</td>
</tr>
<tr>
<td>$\hat{\theta}_{mc1}$</td>
<td>-227.747</td>
<td>713472.8</td>
</tr>
<tr>
<td>$\hat{\theta}_{mc2}$</td>
<td>30.7674</td>
<td>339859.4</td>
</tr>
<tr>
<td>$\hat{\theta}_{mmb}$</td>
<td>1759.951</td>
<td>43417.63</td>
</tr>
<tr>
<td>$\hat{\theta}_{mmb}$</td>
<td>-14.4796</td>
<td>20700.05</td>
</tr>
</tbody>
</table>
Table 3: MSE of Some Estimators and $\hat{t}_{new}$ using Populations 3&4

<table>
<thead>
<tr>
<th>Estimators</th>
<th>Population 3</th>
<th>Population 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIAS</td>
<td>MSE</td>
</tr>
<tr>
<td>$\hat{t}_{mean}$</td>
<td>0</td>
<td>16.44188</td>
</tr>
<tr>
<td>$\hat{t}_{ratio}$</td>
<td>-0.20684</td>
<td>11.77569</td>
</tr>
<tr>
<td>$\hat{t}_{cph}$</td>
<td>-0.00346</td>
<td>11.76569</td>
</tr>
<tr>
<td>$\hat{t}_{ge}$</td>
<td>-0.08028</td>
<td>11.76569</td>
</tr>
<tr>
<td>$\hat{t}_{exp,cph}$</td>
<td>0.1132175</td>
<td>2.413311</td>
</tr>
<tr>
<td>$\hat{t}_{KC1}$</td>
<td>-0.02323</td>
<td>15.17423</td>
</tr>
<tr>
<td>$\hat{t}_{KC2}$</td>
<td>-0.01549</td>
<td>15.59678</td>
</tr>
<tr>
<td>$\hat{t}_{KC3}$</td>
<td>-0.00218</td>
<td>16.01933</td>
</tr>
<tr>
<td>$\hat{t}_{exp,ge}$</td>
<td>59.67895</td>
<td>6.285896</td>
</tr>
<tr>
<td>$\hat{t}_{new}$</td>
<td>-0.39759</td>
<td>2.4001</td>
</tr>
</tbody>
</table>

Tables 2 and 3 showed the results of the empirical study on the BIAS, MSE and PRE of some existing estimators and proposed modified estimators $\hat{t}_{new}$ using data sets of Populations 1, 2, 3 and 4. The results obtained showed that the proposed modified estimator $\hat{t}_{new}$ has the minimum MSEs and highest PREs among other estimators considered in the study. This implies that the proposed modified method $\hat{t}_{new}$ demonstrated high level of efficiency over others and can produce better estimate of population mean in the presence of non-response or missing observation on the average.

V. Conclusion

This study suggested new imputation scheme $\hat{t}_{new}$ as an alternative to Singh and Deo (2003), Prasad (2017), Singh and Horn (2000), Singh et al. (2014). The efficiency of the proposed modified estimator $\hat{t}_{new}$ over other estimators was demonstrated using four (4) sets of populations and the results revealed that the proposed modified estimator $\hat{t}_{new}$ has minimum MSEs and highest PREs. From the results of empirical study, it is concluded that the proposed modified estimator $\hat{t}_{new}$ is recommended for usage in Sample Survey.

VI. References


***