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Power Amarendra Distribution with Properties and Applications

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Abstract: **In this paper we proposed a two-parameter power Amarendra distribution which includes Amarendra distribution as particular case. Its statistical properties including behavior of its probability density function for varying values of parameters, moments, hazard rate function, mean residual life function and stochastic ordering has been discussed. The estimation of the parameters of the distribution has been discussed using maximum likelihood estimation. Applications of the distribution have been investigated using real lifetime datasets from engineering and biomedical sciences.**

Index Terms: **Amarendra distribution,Descriptive measures, Reliability properties,Maximum likelihood estimation,Applications.**

I. INTRODUCTION

Exponential distribution is the most widely used one parameter (scale parameter) lifetime distribution. But the greatest drawback of exponential distribution is the constant hazard rate which limits its applications. In real life there are several datasets where monotonically increasing or decreasing hazard rate has been observed. During recent decades several two-parameter lifetime distributions have been proposed in statistics literature to model monotonically increasing hazard rates. For instance, power Lindley distribution (PLD) by Ghitany et al (2013), power Akash distribution (PAKD) by Shanker and Shukla (2017), power Aradhana distribution (PARD) by Shanker and Shukla (2018) and power Sujatha distribution(PSUD) by Shanker and Shukla (2019), are some among others. It should be noted that PLD is obtained using power transformation from Lindley distribution proposed by Lindley (1958). Similarly, PAKD, PARD and PSUD have been obtained using power transformation from Akash distribution proposed by Shanker (2015), Aradhana distribution suggested by Shanker (2016a) and Sujatha distribution introduced by Shanker (2016b), respectively.

It has been observed that although all these power distributions are useful for monotonically increasing hazard rate datasets, but there are several datasets where these distributions do not provide good fit. In statistics literature, there is one parameter lifetime distribution named Amarendra distribution which provides much closer fit than Lindley, Akash, Aradhana and Sujatha distributions. The main reason behind the study of considering power Amarendra distribution is that as Amarendra distribution gives much closer fit than several one parameter lifetime distributions, it is expected that power Amarendra distribution will provide much closer fit than the power distributions corresponding to these one parameter lifetime distributions.

Shanker (2016c) introduced one parameter lifetime distribution named Amarendra distribution defined by probability density function (pdf) and cumulative distribution function (cdf) as

$$f(x;\theta)=\frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\left(1+x+x^2+x^3\right)e^{-\theta x};\ x>0,\theta>0$$

$$F(x,\theta)=1-\left[1+\frac{\theta^3x^3+\theta^2(\theta+3)x^2+\theta\left(\theta^2+2\theta+6\right)x}{\theta^3+\theta^2+2\theta+6}\right]e^{-\theta x};x>0,\theta>0$$

Amarendra distribution is a mixture of exponential (θ), gamma $(2,\theta)$, gamma $(3,\theta)$, gamma $(4,\theta)$ distributions with mixing proportion $\frac{\theta^3}{\theta^3+\theta^2+2\theta+6}$, $\frac{\theta^2}{\theta^3+\theta^2+2\theta+6}$, $\frac{2\theta}{\theta^3+\theta^2+2\theta+6}$, $\frac{6}{\theta^3+\theta^2+2\theta+6}$ respectively. Statistical properties, estimation of parameter and applications of Amarendra distribution are available in Shanker (2016c)

The main motivation of deriving power Amarendra distribution is to make Amarendra distribution more flexible with an additional shape parameter. The statistical properties including behaviour of its pdf, cdf, hazard function, mean residual life function with varying values of parameters have been discussed. Stochastic ordering of the distribution has been studied and the raw moments and the variance of the distribution have been obtained. Maximum likelihood estimation has been

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discussed to estimate parameters of the distribution. The estimation of the parameters of the distribution has been discussed using maximum likelihood estimation. Finally two examples of real lifetime datasets have been presented to show its applications in engineering and biomedical sciences.

II. Power Amarendra Distribution

Assuming the power transformation $X = Y^{1/\alpha}$ in Amarendra distribution, the pdf of the random variable X can be obtained as

$$f(x;\theta,\alpha) = \frac{\alpha\theta^4}{\theta^3+\theta^2+2\theta+6} x^{\alpha-1}\left(1+x^\alpha+x^{2\alpha}+x^{3\alpha}\right)e^{-\theta x^\alpha}, x>0, \theta>0, \alpha$$

We would call this as power Amarendra distribution (PAMD) with parameters θ and α. Like Amarendra distribution, PAMD is also a convex combination of Weibull (θ,α) distribution, generalized gamma $(2,\alpha,\theta)$ distribution, generalized gamma $(3,\alpha,\theta)$ distribution and generalized gamma $(4,\alpha,\theta)$ distributions. For, we have

$$f(x;\theta,\alpha) = p_1\, g_1(x;\theta,\alpha) + p_2\, g_2(x;\theta,\alpha) + p_3\, g_3(x;\theta,\alpha) + p_4\, g_4(x;\theta,\alpha),$$

where

$$p_1 = \frac{\theta^3}{\theta^3+\theta^2+2\theta+6}$$

$$p_2 = \frac{\theta^2}{\theta^3+\theta^2+2\theta+6}$$

$$p_3 = \frac{2\theta}{\theta^3+\theta^2+2\theta+6}$$

$$p_4 = \frac{6}{\theta^3+\theta^2+2\theta+6}$$

$$g_1(x;\theta,\alpha) = \alpha\theta e^{-\theta x^\alpha} x^{\alpha-1}; x>0, \theta>0, \alpha>0$$

$$g_2(x;\theta,\alpha) = \frac{\alpha\theta^2}{\Gamma(2)} e^{-\theta x^\alpha} x^{2\alpha-1}; x>0, \theta>0, \alpha>0$$

$$g_3(x;\theta,\alpha) = \frac{\alpha\theta^3}{\Gamma(3)} e^{-\theta x^\alpha} x^{3\alpha-1}; x>0, \theta>0, \alpha>0$$

$$g_4(x;\theta,\alpha) = \frac{\alpha\theta^4}{\Gamma(4)} e^{-\theta x^\alpha} x^{4\alpha-1}; x>0, \theta>0, \alpha>0$$

The corresponding cdf of PAMD can be obtained as

$$F(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta^3\left(x^{3\alpha}+x^{2\alpha}+x^\alpha\right) + \theta^2\left(3x^{2\alpha}+2x^\alpha\right) + 6\theta x^\alpha}{\theta^3+\theta^2+2\theta+6}\right] e^{-\theta x^\alpha}, x>0, \theta>0, \alpha>0$$

The pdf and cdf of PAMD for various values of its parameters are presented in figure 1 and figure 2, respectively.

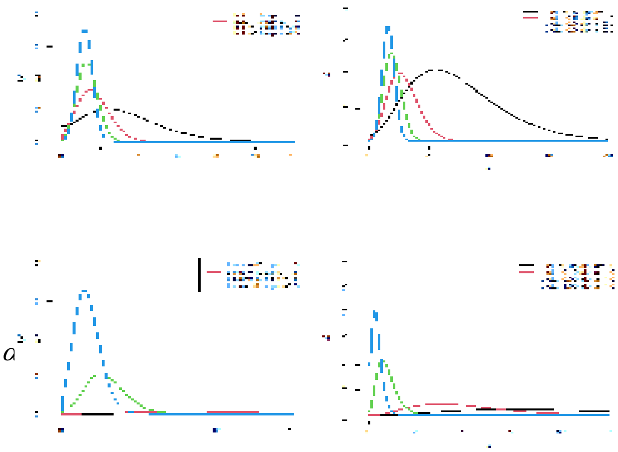


Fig 1: The pdf of PAMD for varying values of parameters (θ,α)

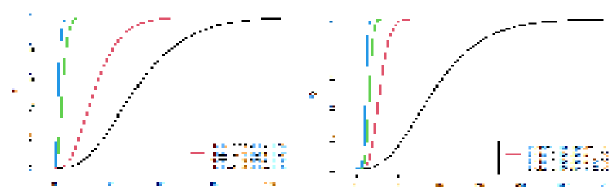


Fig 2: The cdf of PAMD for varying values of parameters (θ,α)

III. Moments of Power amarendra distribution

The rth moment about origin denoted by μ_r' of PAMD can be obtained as

$$\mu_r' = E\left(X^r\right) = \frac{r\Gamma\left(\frac{r}{\alpha}\right)\left\{\alpha^3\theta^3 + (\alpha+r)\alpha^2\theta^2 + (\alpha+r)(2\alpha+r)\alpha\theta + (\alpha+r)(2\alpha+r)(3\alpha+r)\right\}}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{r}{\alpha}}}; r=1,2,3,\ldots$$

Thus the first four moments about origin of the PAMD are thus obtained as

$$\mu_1' = \frac{\left[\alpha^3\theta^3 + (\alpha+1)\left\{\alpha^2\theta^2 + (2\alpha+1)(\alpha\theta+3\alpha+1)\right\}\right]\Gamma\left(\frac{1}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{1}{\alpha}}},$$

$$\mu_2' = \frac{2\left[\alpha^3\theta^3 + (\alpha+2)\left\{\alpha^2\theta^2 + (2\alpha+2)(\alpha\theta+3\alpha+2)\right\}\right]\Gamma\left(\frac{2}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{2}{\alpha}}}$$

$$\mu_3' = \frac{3\left[\alpha^3\theta^3+(\alpha+3)\begin{Bmatrix}\alpha^2\theta^2\\+(2\alpha+3)(\alpha\theta+3\alpha+3)\end{Bmatrix}\right]\Gamma\left(\frac{3}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{3}{\alpha}}}$$

$$\mu_4' = \frac{4\left[\alpha^3\theta^3+(\alpha+4)\begin{Bmatrix}\alpha^2\theta^2\\+(2\alpha+4)(\alpha\theta+3\alpha+4)\end{Bmatrix}\right]\Gamma\left(\frac{4}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{4}{\alpha}}},$$

Therefore, the variance of PAMD can be obtained as

$$\mu_2 = \mu_2' - \left(\mu_1'\right)^2$$

$$= \frac{2\left[\alpha^3\theta^3+(\alpha+2)\begin{Bmatrix}\alpha^2\theta^2+\\(2\alpha+2)(\alpha\theta+3\alpha+2)\end{Bmatrix}\right]\Gamma\left(\frac{2}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{2}{\alpha}}} - \left[\frac{\left[\alpha^3\theta^3+(\alpha+1)\begin{Bmatrix}\alpha^2\theta^2+\\(2\alpha+1)(\alpha\theta+3\alpha+1)\end{Bmatrix}\right]\Gamma\left(\frac{1}{\alpha}\right)}{\alpha^4\left(\theta^3+\theta^2+2\theta+6\right)\theta^{\frac{1}{\alpha}}}\right]^2$$

If required central moments μ_3 and μ_4 can be obtained using the formula

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\left(\mu_1'\right)^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\left(\mu_1'\right)^2 - 3\left(\mu_1'\right)^4$$

IV. Reliability Properties of Power Amarendra Distribution

Survival Function

The survival function of PAMD can be obtained as

$$S(x;\theta,\alpha) = 1 - F(x;\theta,\alpha)$$

$$= \left[1+\frac{\theta^3\left(x^{3\alpha}+x^{2\alpha}+x^{\alpha}\right)+\theta^2\left(3x^{2\alpha}+2x^{\alpha}\right)+6\theta x^{\alpha}}{\theta^3+\theta^2+2\theta+6}\right]e^{-\theta x^{\alpha}}\ ;\ x>0,\theta>0,\alpha>0$$

Hazard Function

The hazard function $h(x;\theta,\alpha)$ of PAMD is obtained as

$$h(x;\theta,\alpha) = \frac{f(x;\theta,\alpha)}{S(x;\theta,\alpha)}$$

$$= \frac{\alpha\theta^4\left(1+x^{\alpha}+x^{2\alpha}+x^{3\alpha}\right)x^{\alpha-1}}{\left[\begin{matrix}\theta^3\left(x^{3\alpha}+x^{2\alpha}+x^{\alpha}\right)+\theta^2\left(3x^{2\alpha}+2x^{\alpha}\right)\\+60\theta x^{\alpha}+\left(\theta^3+\theta^2+2\theta+6\right)\end{matrix}\right]}$$

4.3 Mean Residual Life Function

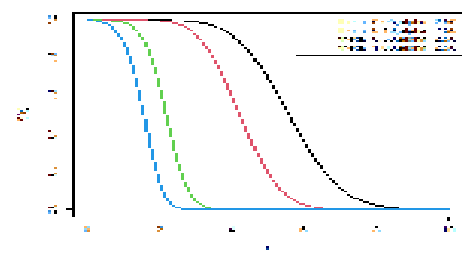
The mean residual function $m(x;\theta,\alpha)$ of PAMD is obtained as

$$m(x;\theta,\alpha) = E(X-x\mid X\geq x)$$

$$= \frac{1}{S(x;\theta,\alpha)}\int_x^{\infty} t\, f(t;\theta,\alpha)\,dt - x$$

$$= \frac{\left(\theta^3\right)\Gamma\left(\frac{1}{\alpha}+1,\theta x^{\alpha}\right)+\theta^2\Gamma\left(\frac{1}{\alpha}+2,\theta x^{\alpha}\right)+\theta\Gamma\left(\frac{1}{\alpha}+3,\theta x^{\alpha}\right)+\Gamma\left(\frac{1}{\alpha}+4,\theta x^{\alpha}\right)}{\theta^{1/\alpha}\left[\theta^3\left(x^{3\alpha}+x^{2\alpha}+x^{\alpha}\right)+\theta^2\left(3x^{2\alpha}+2x^{\alpha}+60x^{\alpha}\right)\right]e^{-\theta x^{\alpha}}} - x$$

The survival, hazard and mean residual life functions of PAMD for various values of its parameters are presented in figure 3, 4 and 5 respectively



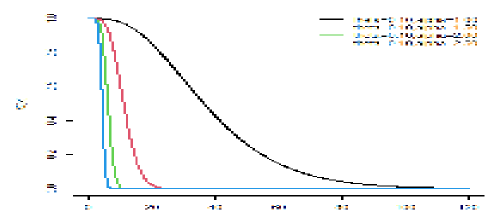


Fig 3: Survival function of PAMD for varying values of parameters (θ,α)

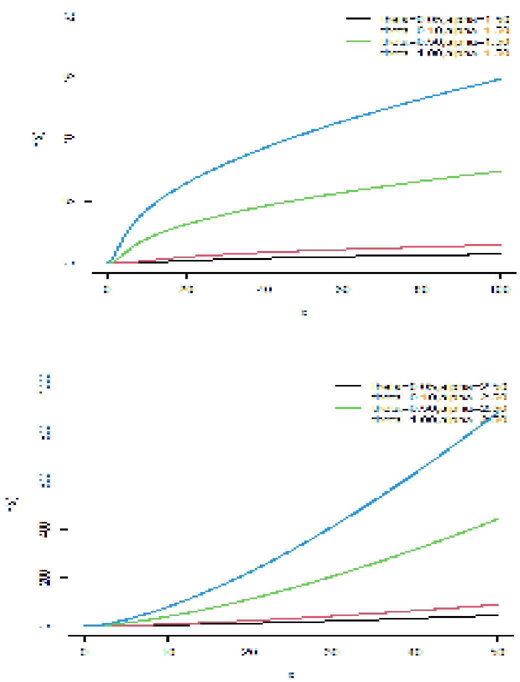


Fig 4: Hazard function of PAMD for varying values of parameters (θ, α)

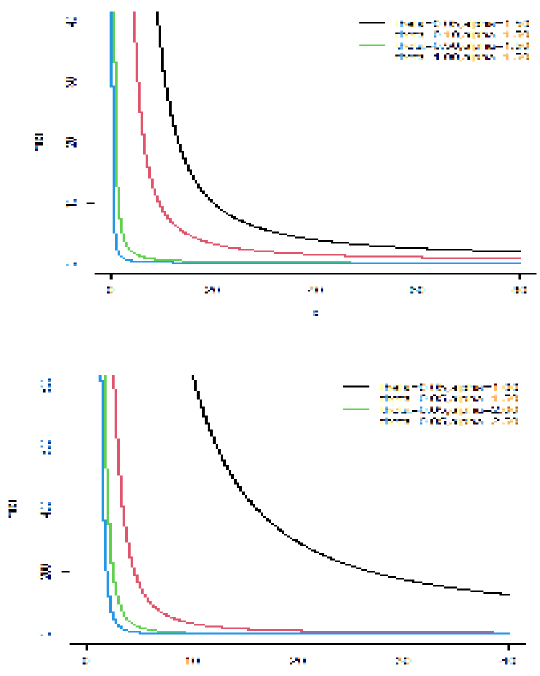


Fig 5: Mean residual life function of PAMD for varying values of parameters (θ, α)

V. Stochastic Orderings

The stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i) Stochastic order $(X \le_{st} Y)$ if $F_X(x) \ge F_Y(x)$ for all x

(ii) Hazard rate order $(X \le_{hr} Y)$ if $h_X(x) \ge h_Y(x)$ for all x

(iii) Mean residual life order $(X \le_{mrl} Y)$ if $m_X(x) \le m_Y(x)$ for all x

(iv) Likelihood ratio order $(X \le_{lr} Y)$ if $\dfrac{f_X(x)}{f_Y(x)}$ decreases in x.

The following interrelationships due to Shaked and Shanthikumar (1994) are well-known for establishing stochastic ordering of distributions

$$\begin{array}{c} X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y \\ \Downarrow \\ X \le_{st} Y \end{array}$$

It can be easily shown that PAMD is ordered with respect to the strongest 'likelihood ratio' ordering. The stochastic ordering of PAMD has been explained in the following theorem:

Theorem: Suppose $X \sim$ PAMD (θ_1, α_1) and $Y \sim$ PAMD (θ_2, α_2). If $\alpha_1 \le \alpha_2$ and $\theta_1 > \theta_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 \ge \theta_2$), then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X(x;\theta_1,\alpha_1)}{f_Y(x;\theta_2,\alpha_2)} = \left[\frac{\alpha_1\theta_1^4\left(\theta_2^3+\theta_2^2+2\theta_2+6\right)}{\alpha_2\theta_2^4\left(\theta_2^3+\theta_1^2+2\theta_1+6\right)}\right] \times \left(\frac{1+x^{\alpha_1}+x^{2\alpha_1}+x^{3\alpha_1}}{1+x^{\alpha_2}+x^{2\alpha_2}+x^{3\alpha_2}}\right) x^{\alpha_1-\alpha_2}\, e^{-\left(\theta_1 x^{\alpha_1}-\theta_2 x^{\alpha_2}\right)}; x>0$$

Now, taking logarithm both sides, we get

$$\log\frac{f_X(x;\theta_1,\alpha_1)}{f_Y(x;\theta_2,\alpha_2)} = \log\left[\frac{\alpha_1\theta_1^4\left(\theta_2^3+\theta_2^2+2\theta_2+6\right)}{\alpha_2\theta_2^4\left(\theta_2^3+\theta_1^2+2\theta_1+6\right)}\right] + \log\left(\frac{1+x^{\alpha_1}+x^{2\alpha_1}+x^{3\alpha_1}}{1+x^{\alpha_2}+x^{2\alpha_2}+x^{3\alpha_2}}\right) + (\alpha_1-\alpha_2)\log x - \left(\theta_1 x^{x^{\alpha_1}} - \theta_2 x^{\alpha_2}\right)$$

This gives

$$\frac{d}{dx}\log\frac{f_X(x;\theta_1,\alpha_1)}{f_Y(x;\theta_2,\alpha_2)} = \frac{\alpha_1-\alpha_2}{x} + \frac{\alpha_1 x^{\alpha_1-1}+2\alpha_1 x^{2\alpha_1-1}+3\alpha_1 x^{3\alpha_1-1}}{1+x^{\alpha_1}+x^{2\alpha_1}+x^{3\alpha_1}} - \frac{\alpha_1 x^{\alpha_2-1}+2\alpha_1 x^{2\alpha_2-1}+3\alpha_1 x^{3\alpha_2-1}}{1+x^{\alpha_2}+x^{2\alpha_2}+x^{3\alpha_2}} - \left(\theta_1\alpha_1 x^{\alpha_1-1} - \theta_2\alpha_2 x^{\alpha_2-1}\right)$$

.

Thus, for $\alpha_1 \le \alpha_2$ and $\theta_1 > \theta_2$ (or $\alpha_1 < \alpha_2$ and $\theta_1 \ge \theta_2$), $\frac{d}{dx}\log\frac{f_X(x;\theta_1,\alpha_1)}{f_Y(x;\theta_2,\alpha_2)} < 0$. This means that $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

VI. Maximum likelihood estimation of parameters of power Amarendra distribution

Suppose $(x_1, x_2, ..., x_n)$ be a random sample of size n from PAMD (θ, α) The log-likelihood function of PAMD can be expressed as

$$\log L = \sum_{i=1}^{n} \log f(x_i; \theta, \alpha)$$

$$= n\log\alpha + 4n\log\theta - n\log(\theta^3 + \theta^2 + 2\theta + 6)$$

$$+(\alpha - 1)\sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\log(1 + x_i^{\alpha} + x_i^{2\alpha} + x_i^{3\alpha}) - \theta\sum_{i=1}^{n}x_i^{\alpha}$$

The maximum likelihood estimates $(\hat{\theta}, \hat{\alpha})$ of parameters (θ, α) of PAMD are the solution of the following log-likelihood equations

$$\frac{\partial \log L}{\partial \theta} = \frac{4n}{\theta} - \frac{n(3\theta^2 + 2\theta + 2)}{\theta^3 + \theta^2 + 2\theta + 6} - \sum_{i=1}^{n} x_i^{\alpha} = 0$$

$$\frac{\partial \log L}{\partial \alpha} = 0$$

$$= \frac{n}{\alpha} + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i + 2x_i^{2\alpha}\log x_i + 3x_i^{3\alpha}\log x_i}{1 + x_i^{\alpha} + x_i^{2\alpha} + x_i^{3\alpha}}$$

$$-\theta\sum_{i=1}^{n}x_i^{\alpha}\log(x_i) = 0$$

These two natural log-likelihood equations do not seem to be solved directly, because they cannot be expressed in closed forms. The (MLE's) $\hat{\theta}$ of (θ, α) can be either computed directly by solving the log-likelihood equation using Newton-Raphson iteration available in R-software till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained or using Fishers scoring method.

As Fisher's scoring method is based on the second order partial derivatives, we have

$$\frac{\partial^2 \log L}{\partial \theta^2} = \frac{-4n}{\theta^2} - \frac{n(15\theta^4 + 8\theta^3 + 10\theta^2 + 8)}{(\theta^3 + \theta^2 + 2\theta + 6)^2}$$

$$\frac{\partial^2 \log L}{\partial \theta \partial \alpha} = -\sum_{i=1}^{n} x_i^{\alpha} \log x_i = \frac{\partial^2 \log L}{\partial \alpha \partial \theta}$$

$$\frac{\partial^2 \log L}{\partial \alpha^2}$$

$$= -\frac{n}{\alpha^2} + \sum_{i=1}^{n}\frac{\left[\begin{array}{l}\left\{x_i^{\alpha}(\log x_i)^2 + 4x_i^{2\alpha}(\log x_i)^2 + 9x_i^{3\alpha}(\log x_i)^2\right\} \\ \times\left(1 + x_i^{\alpha} + x_i^{2\alpha} + x_i^{3\alpha}\right) \\ -\left(x_i^{\alpha}\log x_i + 2x_i^{2\alpha}\log x_i + 3x_i^{3\alpha}\log x_i\right)^2\end{array}\right]}{\left(1 + x_i^{\alpha} + x_i^{2\alpha} + x_i^{3\alpha}\right)^2}$$

The following equation can be solved for MLE's of $\hat{\theta}$ and $\hat{\alpha}$ of PAMD

$$\begin{pmatrix} \frac{\partial^2 \log(\theta,\alpha)}{\partial \theta^2} & \frac{\partial^2 \log(\theta,\alpha)}{\partial \theta \partial \alpha} \\ \frac{\partial^2 \log(\theta,\alpha)}{\partial \alpha \partial \theta} & \frac{\partial^2 \log(\theta,\alpha)}{\partial \alpha^2} \end{pmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{pmatrix} \hat{\theta} - \theta_0 \\ \hat{\alpha} - \alpha_0 \end{pmatrix} = \begin{pmatrix} \frac{\partial \log(\theta,\alpha)}{\partial \alpha} \\ \frac{\partial \log(\theta,\alpha)}{\partial \beta} \end{pmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}},$$

Where θ_0 and α_0 are initial value of θ and α respectively. The initial values of the parameters taken in this paper for estimating parameters are $\theta_0 = 0.5$ and $\alpha_0 = 0.5$.

VII. Goodness of fit

In this section, the goodness of fit of PAMD using maximum likelihood estimates of parameters has been discussed with two real datasets. The goodness of fit has been compared with other two-parameter lifetime distributions. The following two datasets, one from engineering and one from medical science have been considered for testing the goodness of fit of PAMD over other two-parameter lifetime distributions.

Data Set 1: The data set is from Lawless (1982). The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are.

17.88, 28.92, 33, 41.5, 42.12, 45.6, 48.8, 51.8, 52, 54.12, 55.56, 67.8, 68.44, 68.64, 68.9, 84.1, 93.12, 98.6, 105, 106, 128, 128, 173.4

Data Set 2: The data set reported by Efron (1988) represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

6.53 ,7 ,10.42, 14.48, 16.1, 22.7, 34, 41.55, 42, 45.28, 49.4, 53.62, 63 ,64 ,83, 84, 91, 108, 112, 129, 133, 133, 139, 140, 140, 146, 149 ,154, 157, 160, 160, 165, 146, 149, 154, 157 ,160, 160, 165, 173, 176, 218, 225, 241, 248, 273, 277, 297, 405 ,417, 420, 440, 523, 583, 594, 1101, 1146, 1417

In order to compare the considered distributions, values of MLE$(\hat{\theta}, \hat{\alpha})$ along with their

Standard errors, $-2\log L$, AIC (Akaike Information Criterion), K-S (Kolmogorov-Smirnov) Statistics and p-values for the real lifetime dataset 1and dataset 2 has been computed and presented in table1 and table 2 . The formulae for computing AIC and K-S Statistics are as follows:

$AIC = -2logL + 2k$, $K - S = \underset{x}{Sup}\,|F_n(x) - F_0(x)$,

where $k =$ number of parameter, $n =$ sample size . The best distribution is the distribution corresponding to lower values of $-2logL$, AIC and K-S. the fitted plots of the considered distributions to the datasets 1 and 2 are shown in the figure 6.

We compared PAMD to the other two-parameter distribution whose pdf and cdf are mentioned in the given below

Power Lindley Distribution (PLD):

$$f(x;\theta,\alpha) = \frac{\alpha\theta^2}{\theta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\theta x^{\alpha}};\ x>0, \theta>0, \alpha>0$$

$$F(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta}{\theta+1}x^{\alpha}\right]e^{-\theta x^{\alpha}};\ x>0, \theta>0\ , \alpha>0$$

Power Akash Distribution (PAKD):

$$f\left(x;\theta,\alpha\right)=\frac{\alpha\theta^{3}}{\theta^{2}+2}\left(1+x^{2\alpha}\right)x^{\alpha-1}e^{-\theta x^{\alpha}};\;\; x>0,\theta>0,\alpha>0$$

$$F\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^{\alpha}\left(\theta x^{\alpha}+2\right)}{\theta^{2}+2}\right]e^{-\theta x^{\alpha}};\quad x>0,\theta>0\;,\alpha>0$$

Power Sujatha Distribution (PSUD):

$$f\left(x;\theta,\alpha\right)=\frac{\alpha\theta^{3}}{\theta^{2}+\theta+2}\left(1+x^{\alpha}+x^{2\alpha}\right)x^{\alpha-1}e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0$$

(23)

$$F\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^{\alpha}\left(\theta x^{\alpha}+\theta+2\right)}{\theta^{2}+\theta+2}\right]e^{-\theta x^{\alpha}};x>0,\theta>0\;,\alpha>0$$

Power Aradhana Distribution (PARD):

$$f\left(x;\theta,\alpha\right)=\frac{\alpha\theta^{3}x^{\alpha-1}}{\theta^{2}+2\theta+2}\left(1+2x^{\alpha}+x^{2\alpha}\right)e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0$$

$$F\left(x;\theta,\alpha\right)=1-\left[1+\frac{\theta x^{\alpha}\left(\theta x^{\alpha}+2\theta+2\right)}{\theta^{2}+2\theta+2}\right]e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0$$

Table1: ML estimates, *-2logL* , $S.E\left(\hat{\theta},\hat{\alpha}\right)$, AIC, K-S statistic, and P-value of the fitted distribution of dataset1.

Distribution	ML estimate $\hat{\theta}$(S.E of $\hat{\theta}$) $\hat{\alpha}$(S.E of $\hat{\alpha}$)	*-2logL*	AIC	K-S	P-value
PAMD	0.05(0.03) 1.00(0.14)	**226.07**	**230.07**	**0.11**	**0.93**
PLD	0.01(0.09) 1.23(0.20)	227.49	231.49	0.15	0.68
PAKD	0.02(0.01) 1.15(0.16)	226.17	230.17	0.14	0.74
PSUD	0.02(0.01) 1.15(0.15)	226.19	230.19	0.14	0.77
PARD	0.02(0.01) 1.16(0.15)	226.21	230.21	0.18	0.46

Table2: ML estimates, *-2logL* , $S.E\left(\hat{\theta},\hat{\alpha}\right)$, AIC, K-S statistic, and P-value of the fitted distribution of dataset 2.

Distribution	ML estimate $\hat{\theta}$(S.E of $\hat{\theta}$) $\hat{\alpha}$(S.E of $\hat{\alpha}$)	*-2logL*	AIC	K-S	P-value
PAMD	0.30(0.06) 0.49(0.04)	**741.92**	**745.92**	**0.15**	**0.14**
PLD	0.04(0.01) 0.69(0.06)	742.77	746.77	0.17	0.07
PAKD	0.16(0.04) 0.55(0.04)	742.11	746.11	0.18	0.05
PSUD	0.15(0.04) 0.56(0.04)	742.15	746.15	0.19	0.03
PARD	0.14(0.03) 0.57(0.04)	742.18	746.18	0.17	0.07

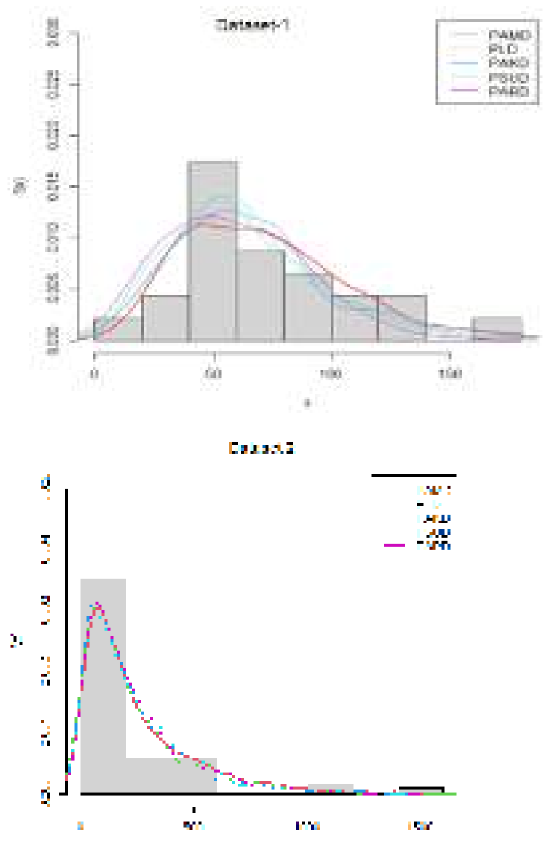


Fig.6: Fitted plots of the considered distributions for dataset 1 and 2.

Based on the values of *-2logL* , AIC, K-S statistic, and the fitted plots of the distribution we can conclude that the PAMD provides most satisfactory fit over other considered distributions

VIII. Concluding remarks

A two-parameter continuous Power Amarendra distribution (PAMD) has been introduced which includes Amarendra distribution proposed by Shanker (2016c) as a particular case. The statistical and reliability properties including shapes of density for varying values of parameters, the moments about the origin, the variance, survival function, hazard function, mean residual function of PAMD have been discussed. The method of maximum likelihood for estimating the parameters has been discussed. Finally, the goodness of fit of PAMD has been discussed with two real lifetime datasets and the fit has been found quite satisfactory as compared with power Lindley distribution (PLD), power Akash distribution (PAKD), power Sujatha distribution (PSUD) and power Aradhana distribution. Therefore, PAMD can be considered as an important lifetime distribution for modeling lifetime data from engineering and biomedical sciences.

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